DATAMAN
MOBILE COMPUTING LABORATORY

Error Characteristics of APS (Ad Hoc Positioning Systems)

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motivation: ad hoc sensor networks

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- a sensor reports a phenomenon and its:
  - position
    - place it on a map
    - routing with small or no routing tables
  - orientation
    - remote navigation
    - fine grained control – camera orientation
- possible solutions
  - GPS + digital compass in each node

related work

- centralized
  - convex optimization [Doherty00]
  - multidimensional scaling [Shang03]
- infrastructure based
  - Grid based [Bulusu00]
  - Cricket, Cricket Compass [Priyantha01]
  - RADAR [Bahl00]
- distributed
  - SPA [Capkun01]
  - AhLOS [Savvides01]

terminology and assumptions ☞

- terminology
  - **landmark** - a node knowing its position
  - **bearing/AoA** - angle w.r.t. some objective
  - **range** - measured distance to some objective
- assumptions
  - no additional infrastructure
  - random deployment, low landmark ratio
  - distributed / localized operation
  - capabilities: AoA, ranging, none

node capabilities

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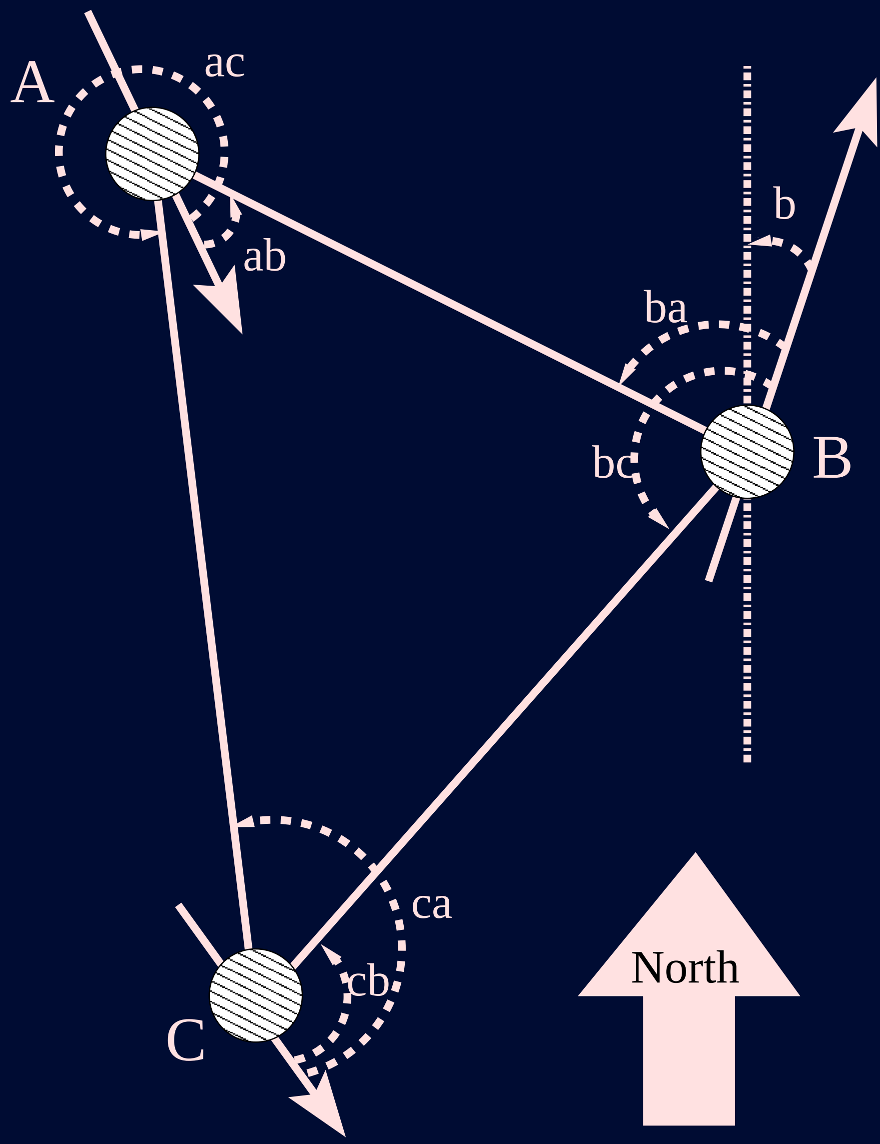

- **Ranging:**
  - **signal strength**
  - **ultrasound**
  - **UWB**
- **AoA:**
  - **Cricket[Pryiantha01]**
  - 5^o **error for** $\pm 40^o$
  - **TDOA based**

APS algorithms

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	Capabilities
DV-hop	**none**
DV-distance	**ranging**
Euclidean	**ranging**
DV-Bearing	**AoA**
DV-radial	**AoA+Compass**
DV-position	**ranging+AoA/Compass**

summary ☞

- motivation
  related work
- terminology
  node capabilities

- APS - algorithm outline
  - example
  - one hop positioning
- *DV-hop*
  - error
  - trilateration error
  - range error
  - position error
- *DV-position*
  - error, simulation
  - *DV-radial* and *Euclidean*
  - parameter space
- future work
- summary
- extra slides

single hop positioning

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- trilateration
  - distances to three known points
- triangulation
  - angles between three known points
- V.O.R.
  - angles from three known points

APS algorithms outline

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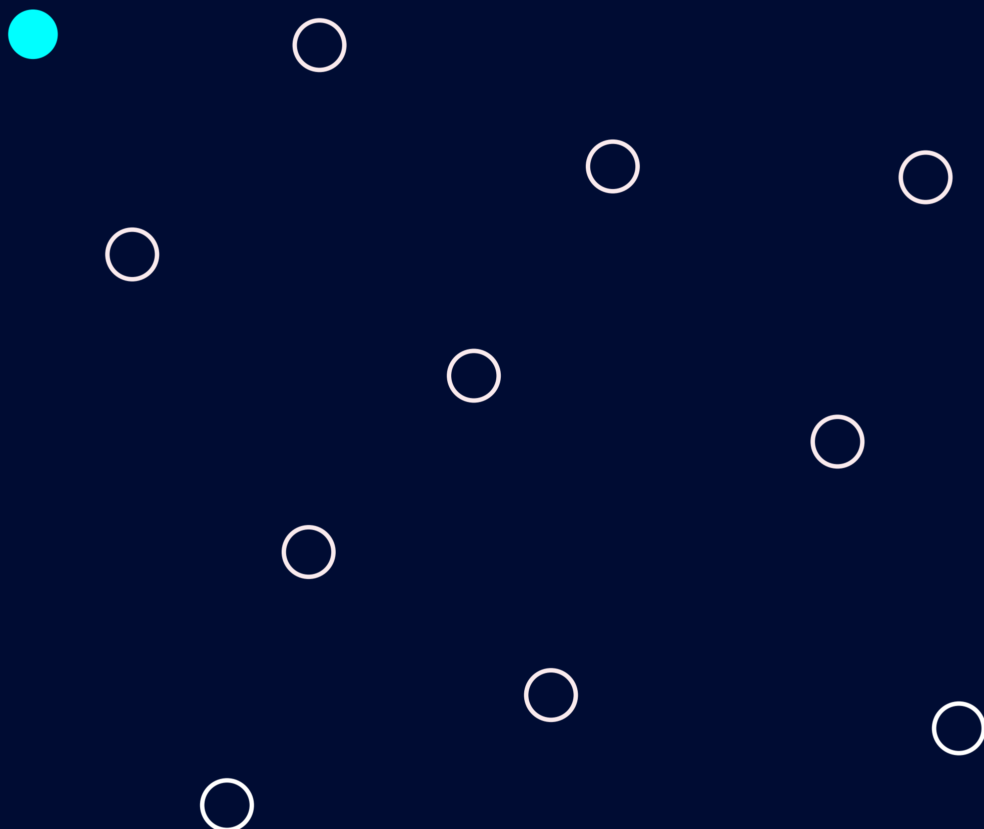
- **landmarks know their position**
- regular nodes
  - find ranges/bearings to immediate neighbors
  - **induction** → infer ranges/bearings to faraway landmarks
- like in DV,
  - ranges/bearings are propagated **hop by hop**
  - each landmark is treated independently

APS example



APS example

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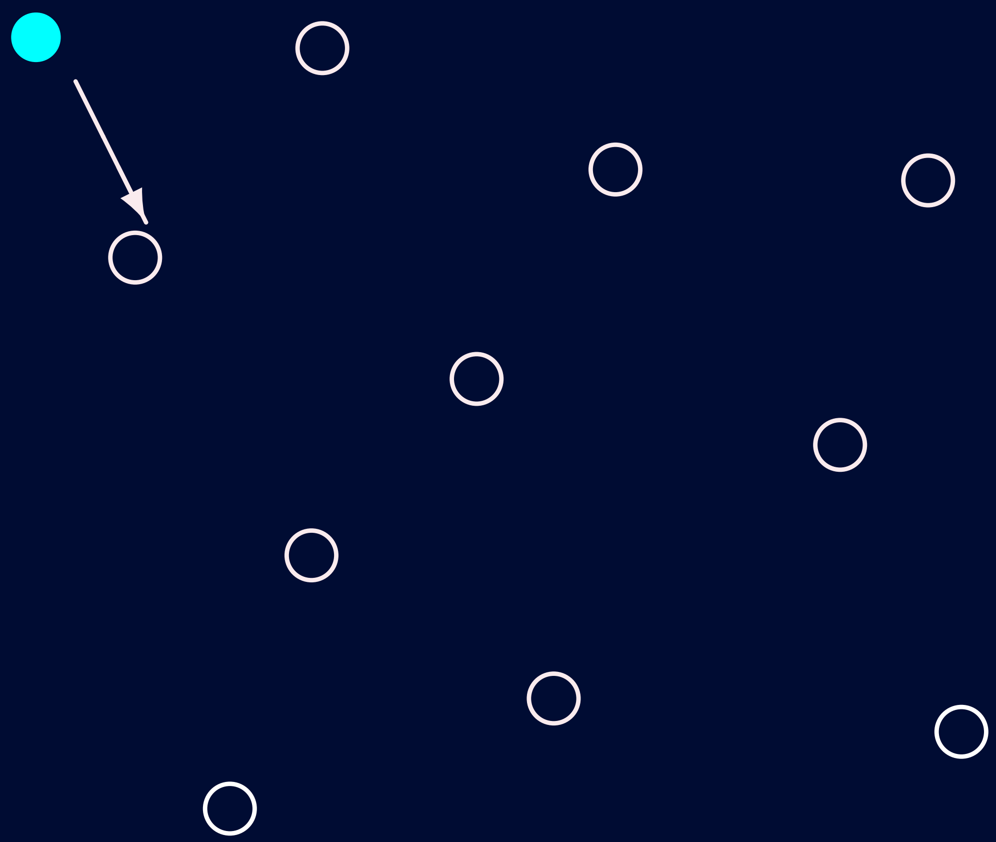

APS example

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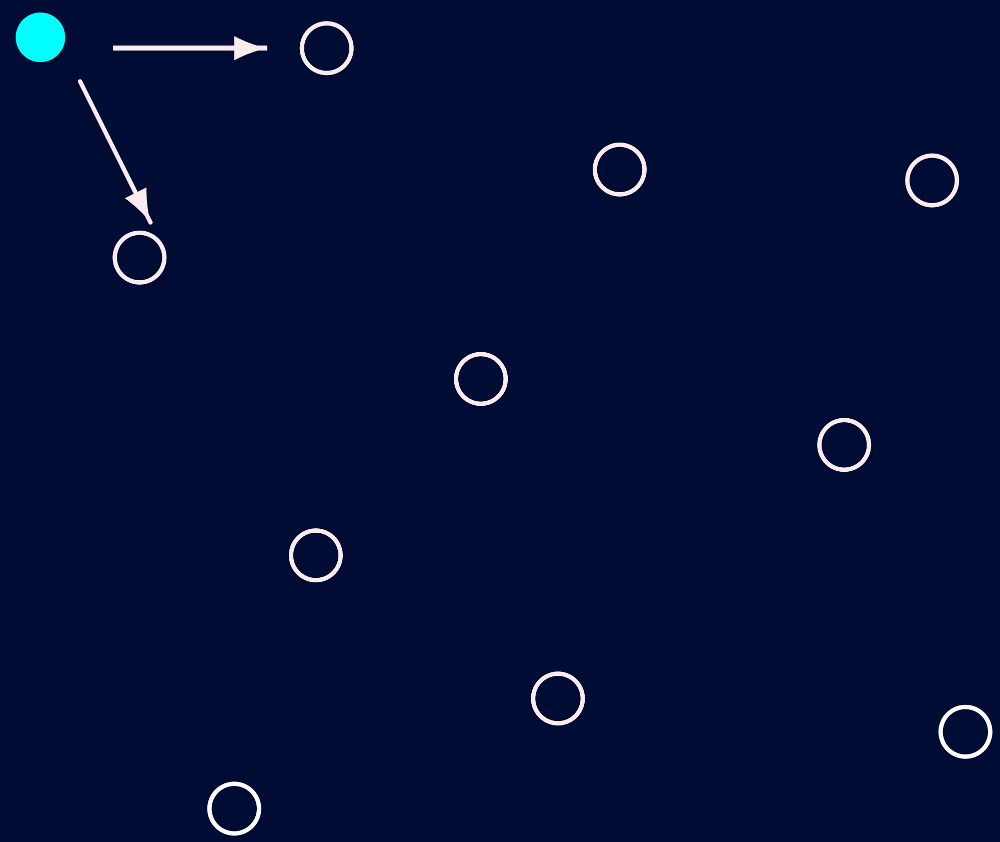

APS example

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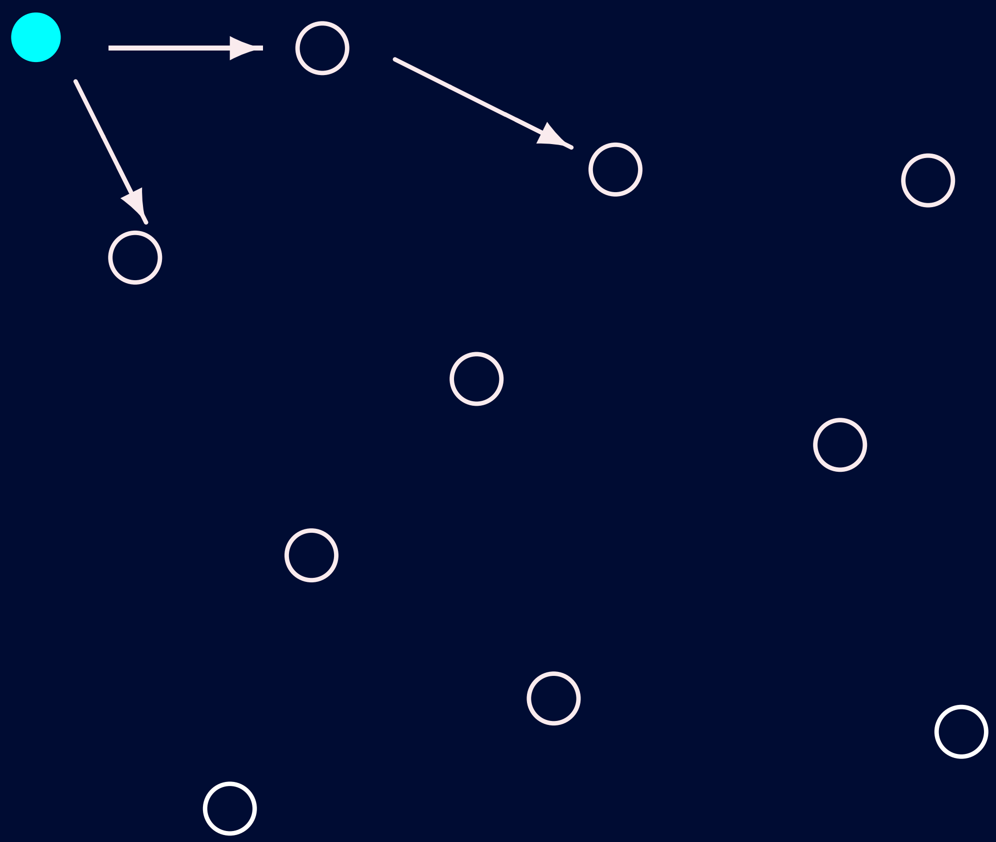

APS example

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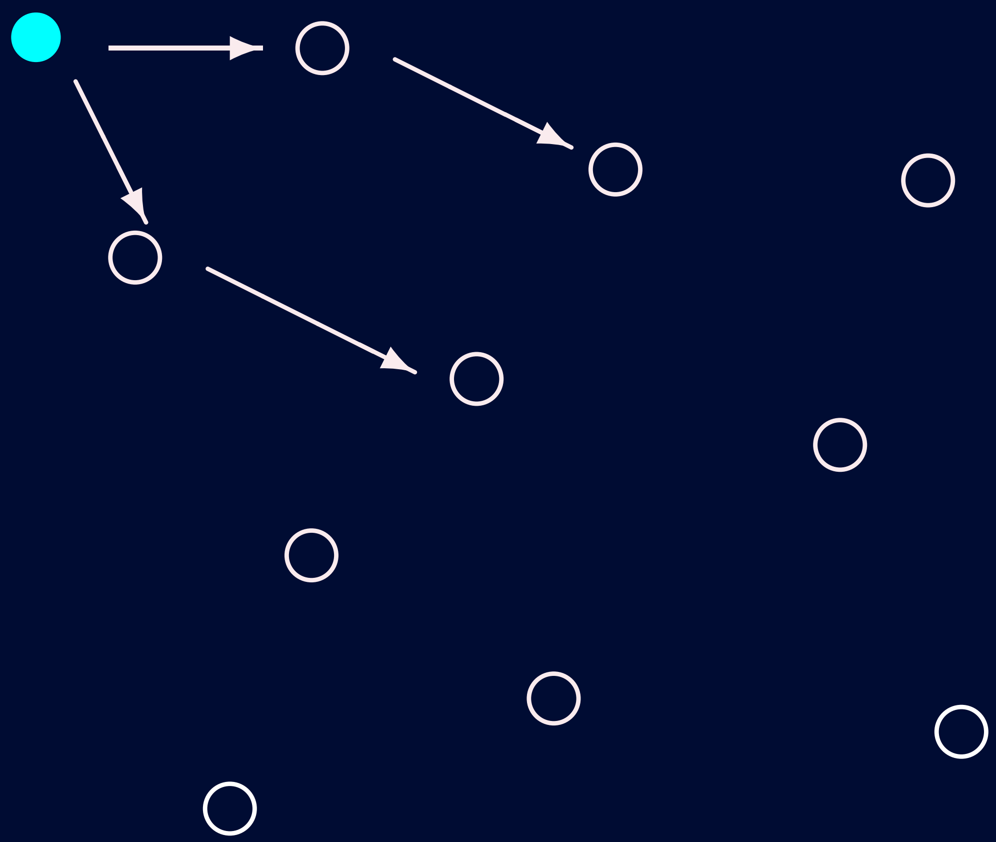

APS example

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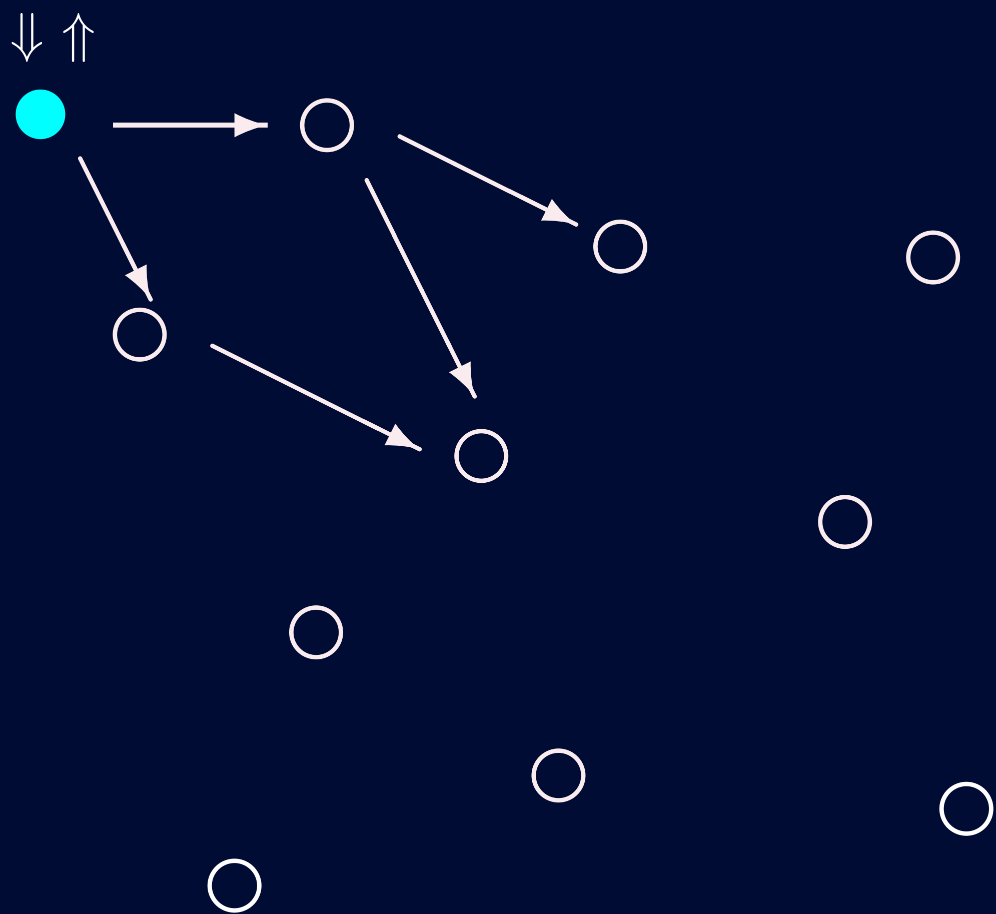

APS example

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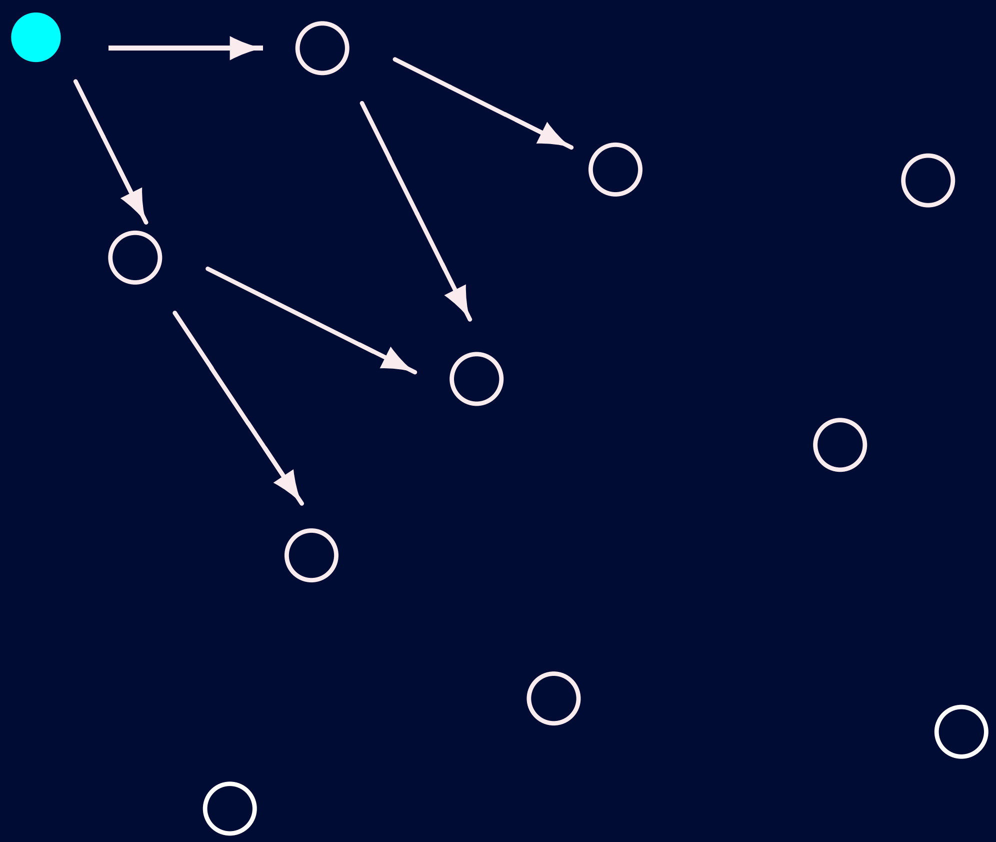

APS example

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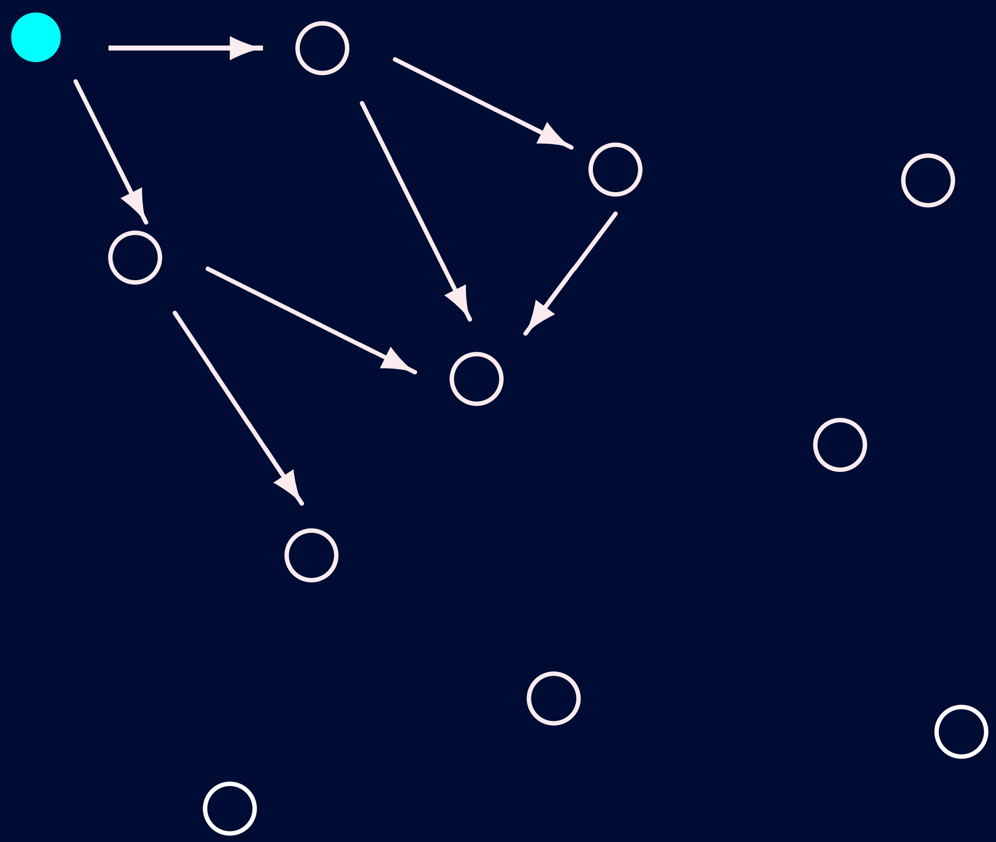

APS example

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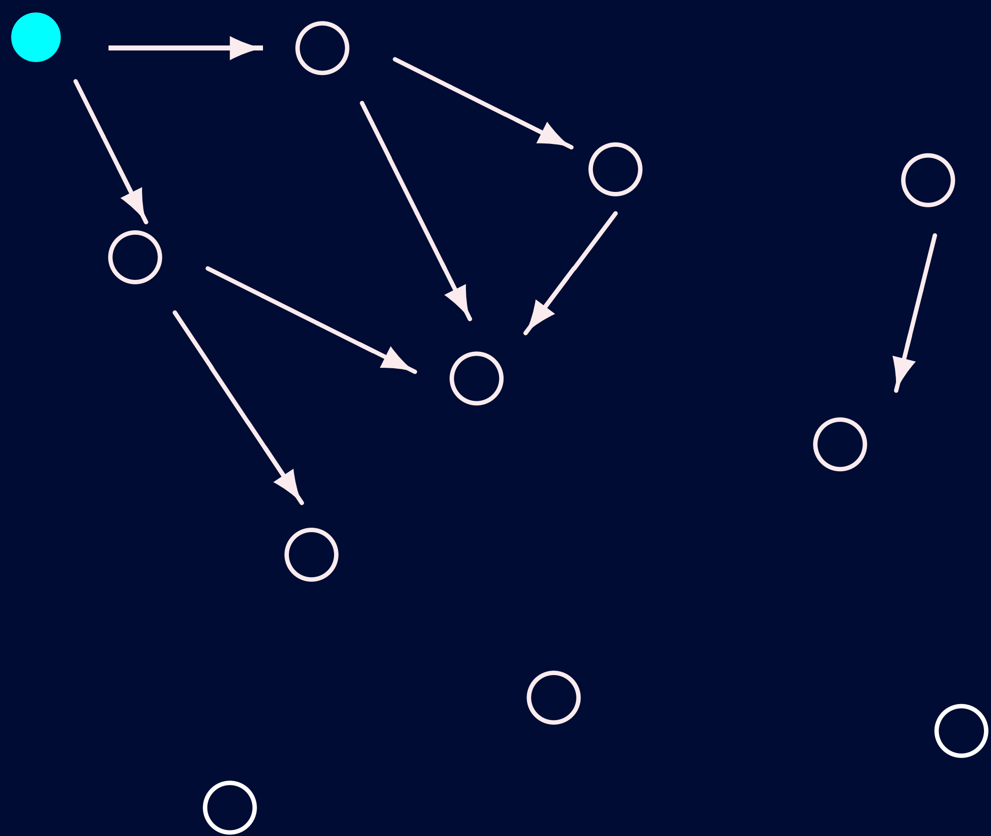

APS example

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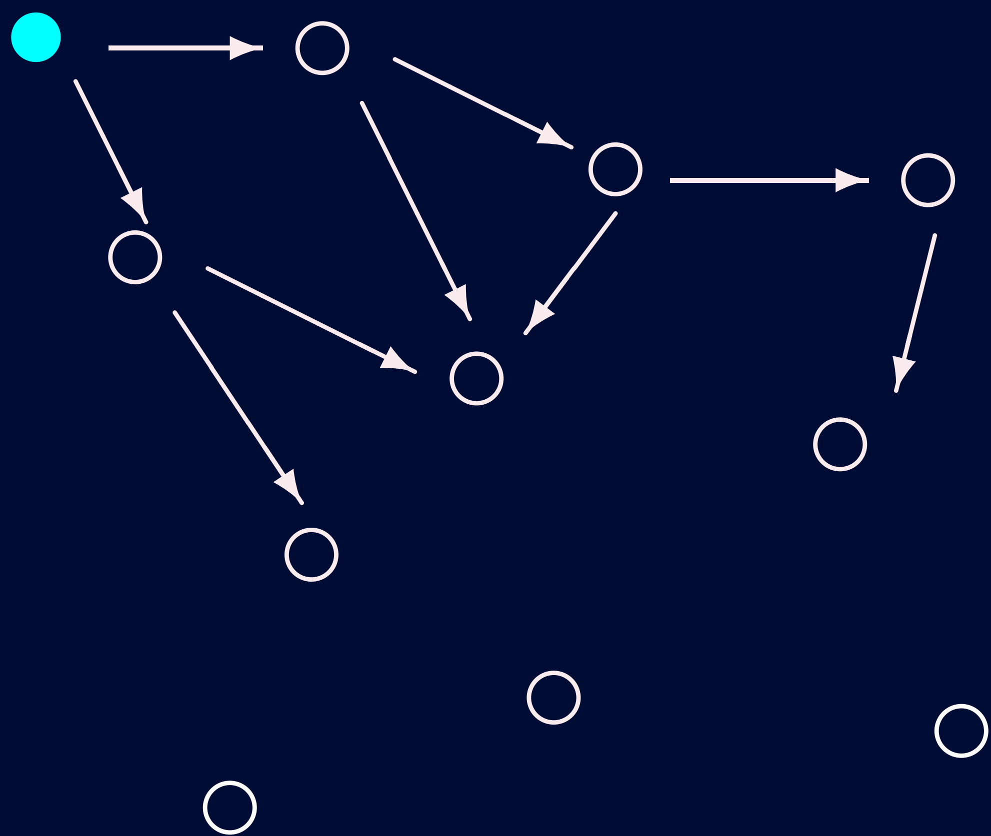

APS example

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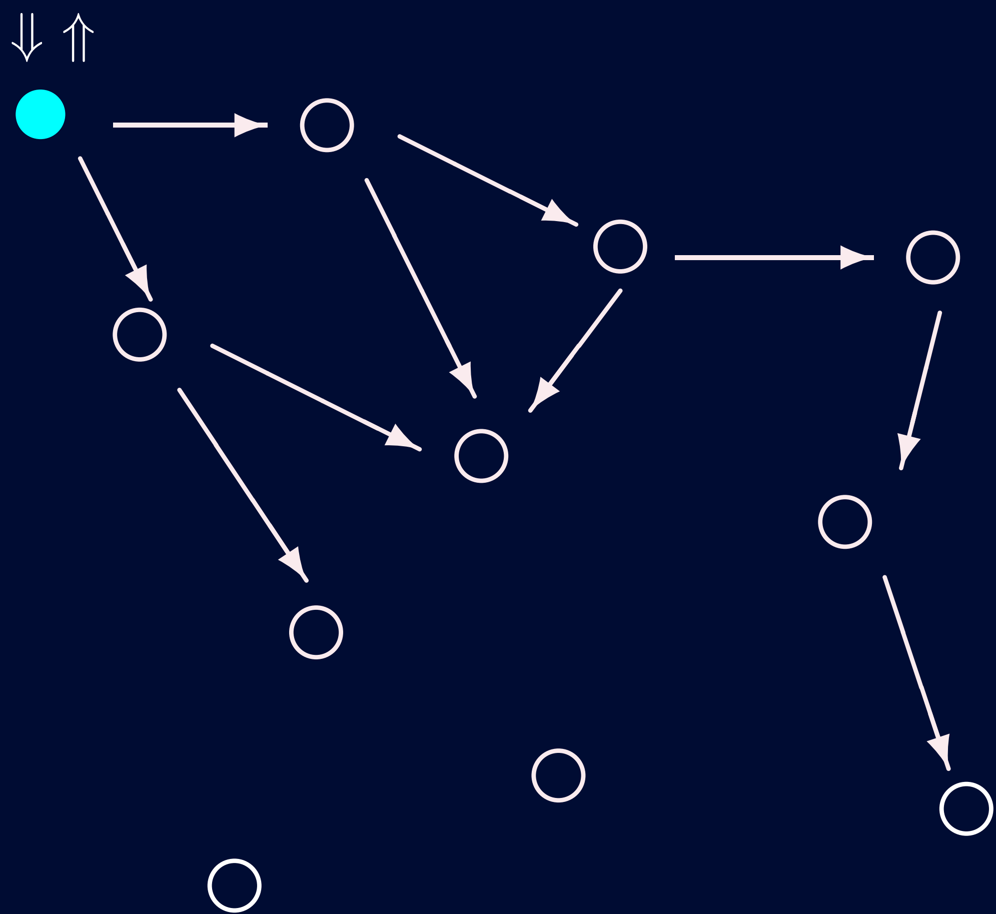

APS example

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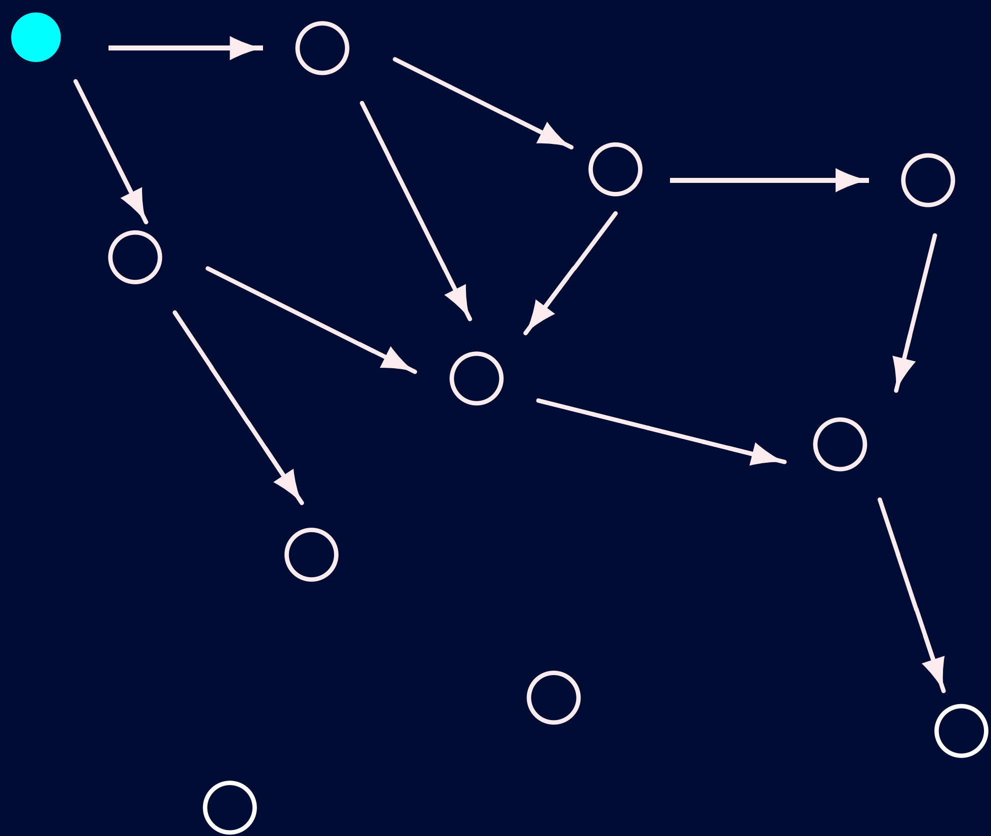

APS example

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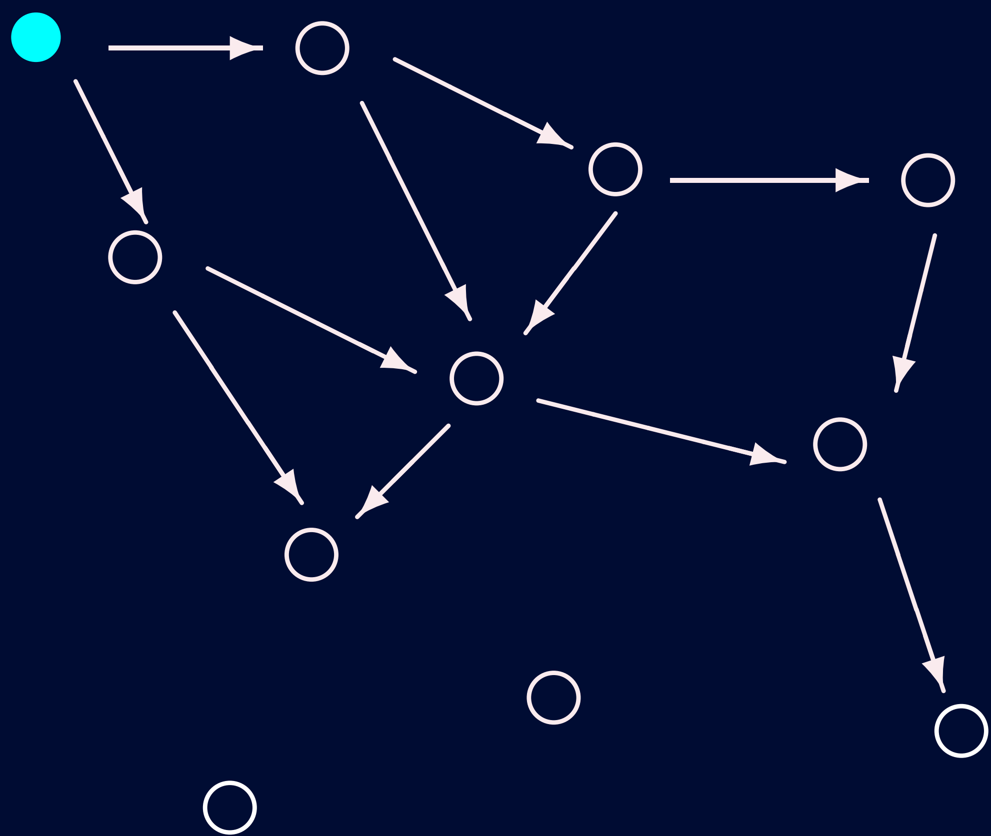

APS example

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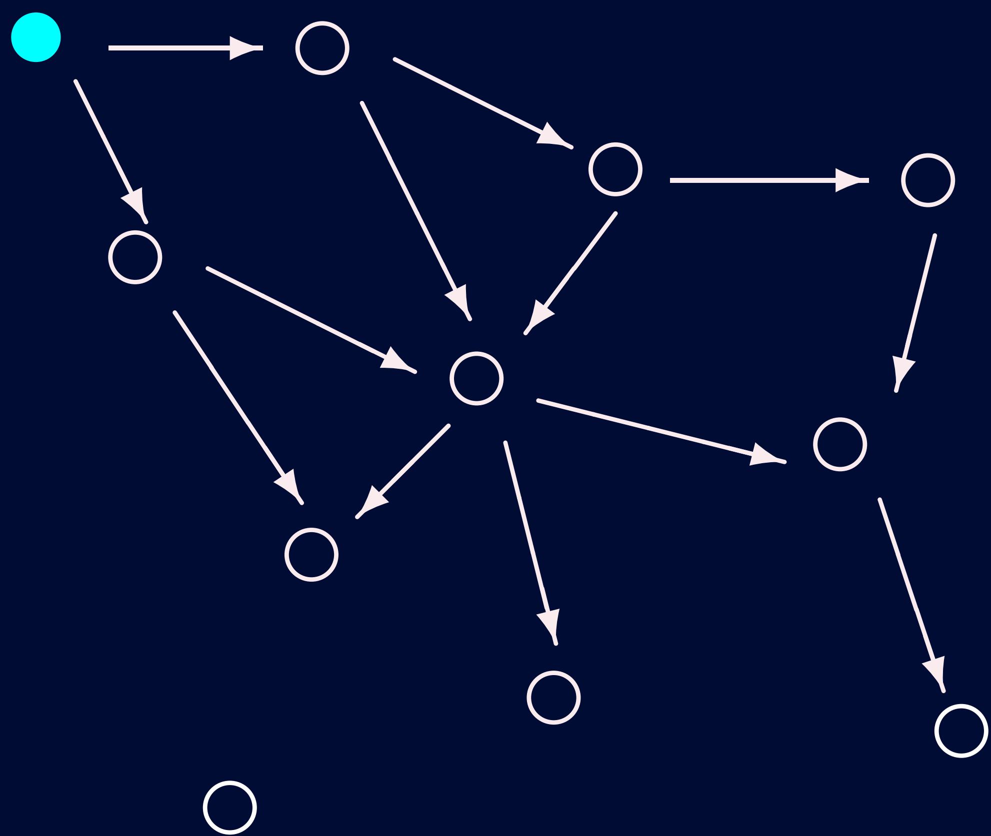

APS example

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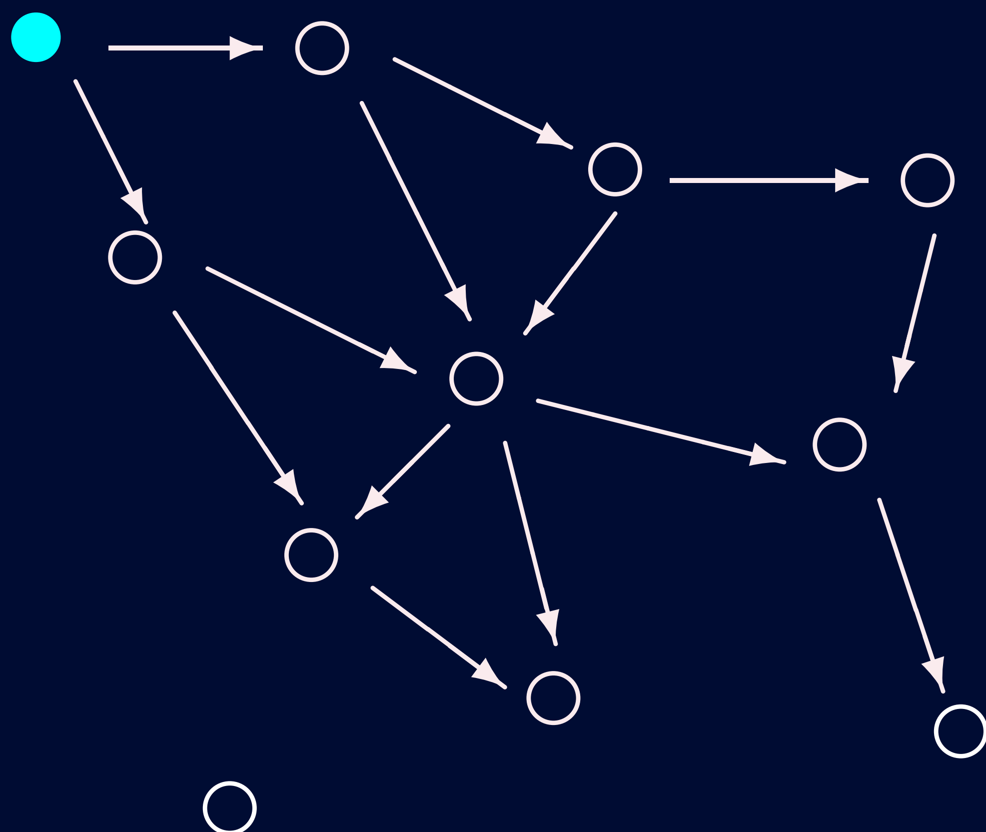

APS example

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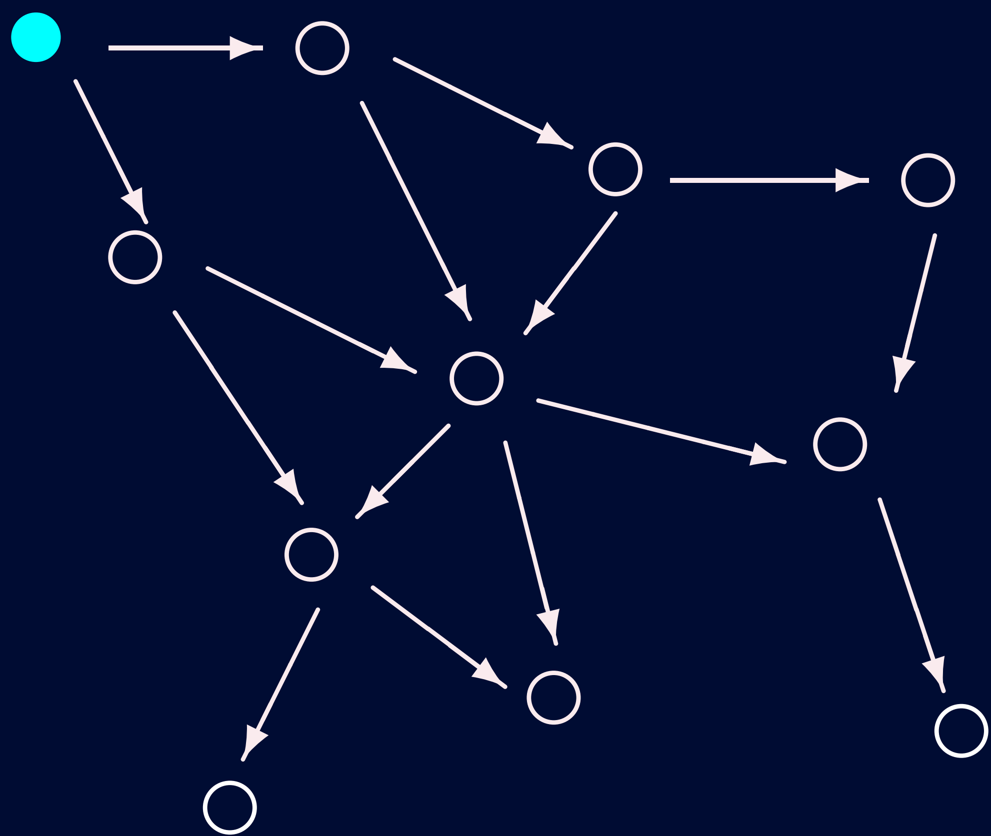

APS example

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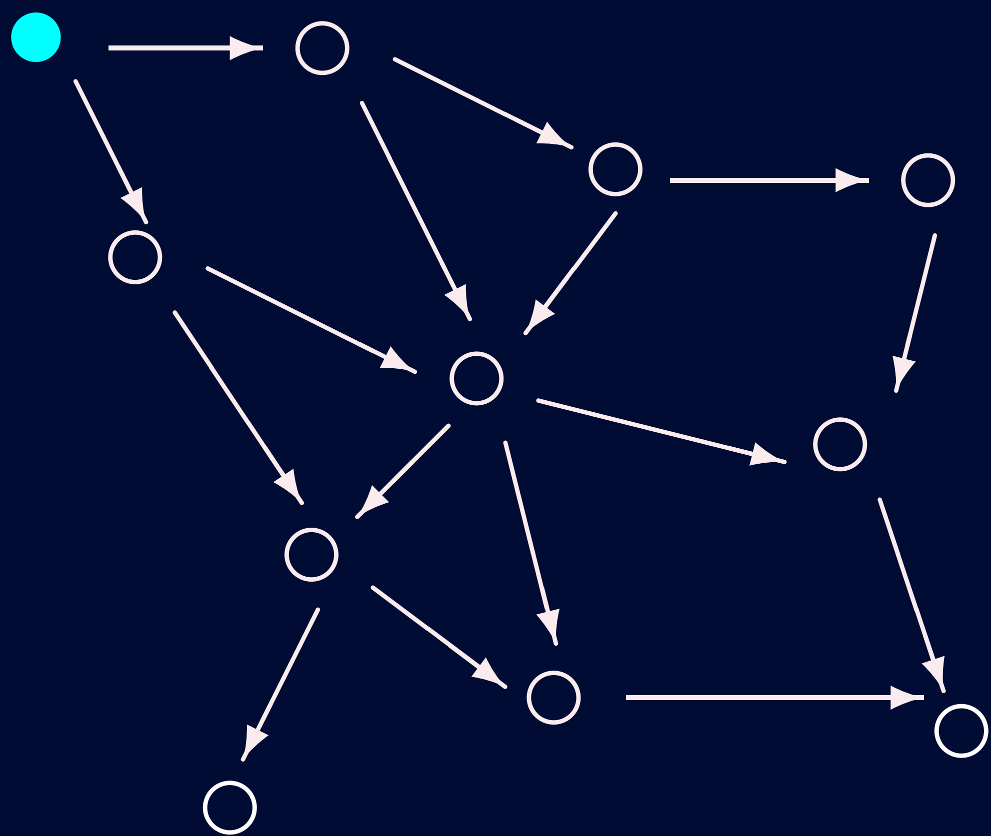

APS example

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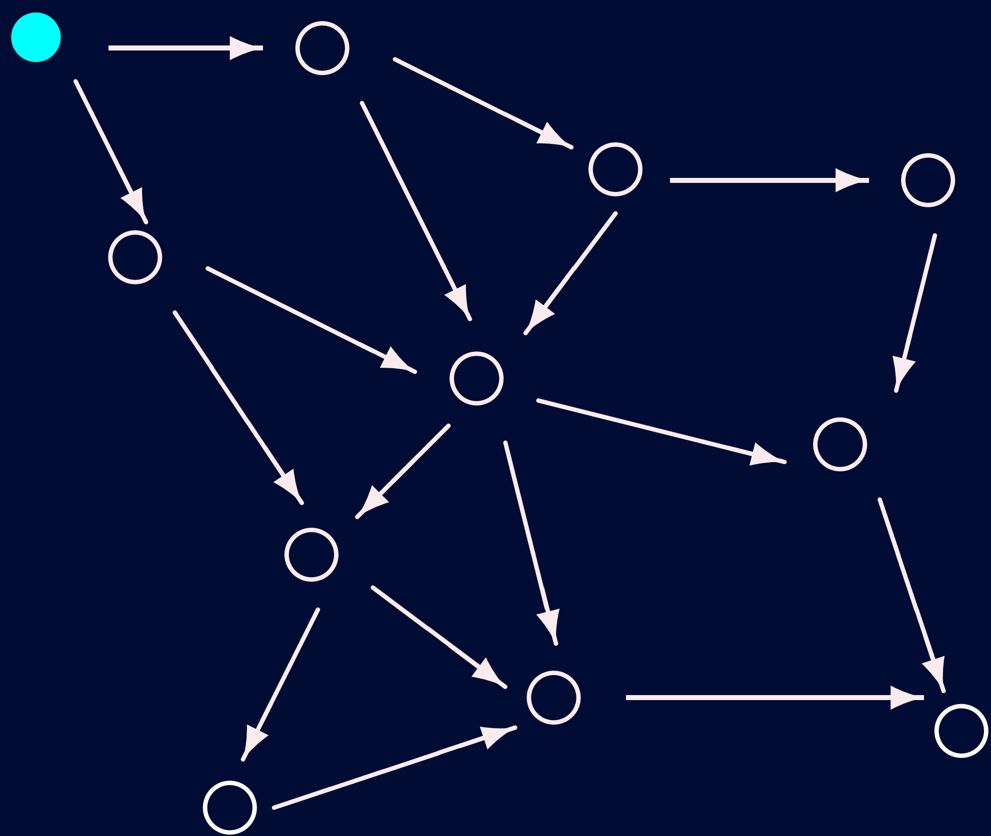

APS example

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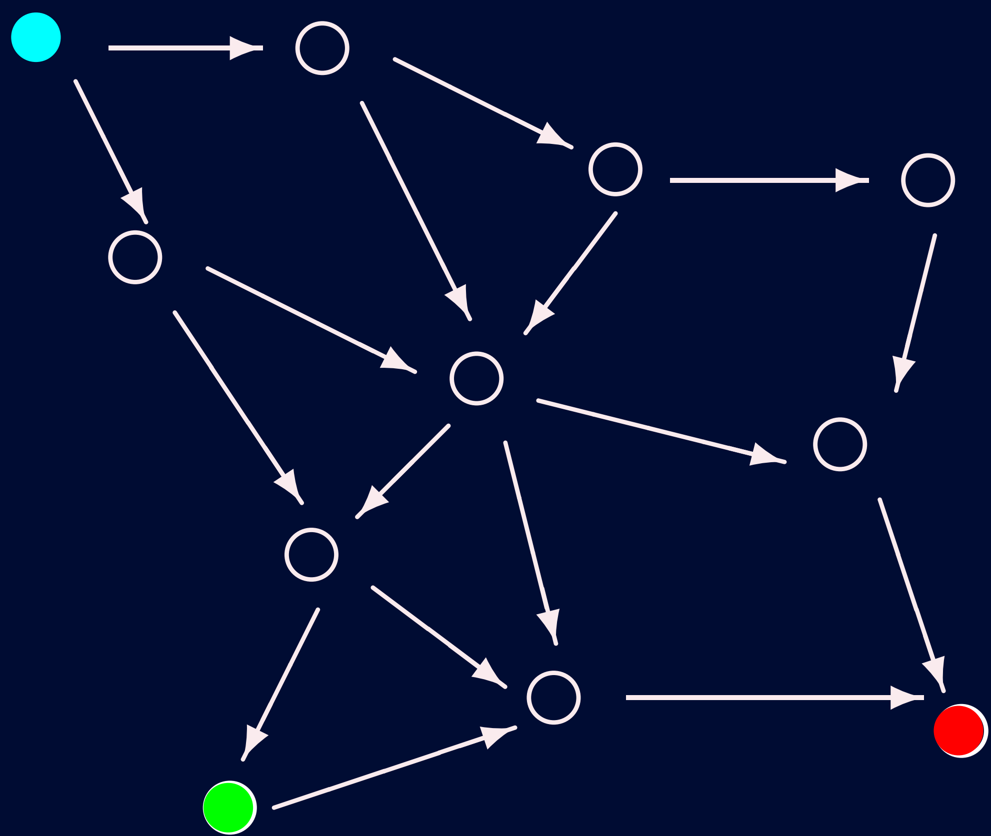

APS example

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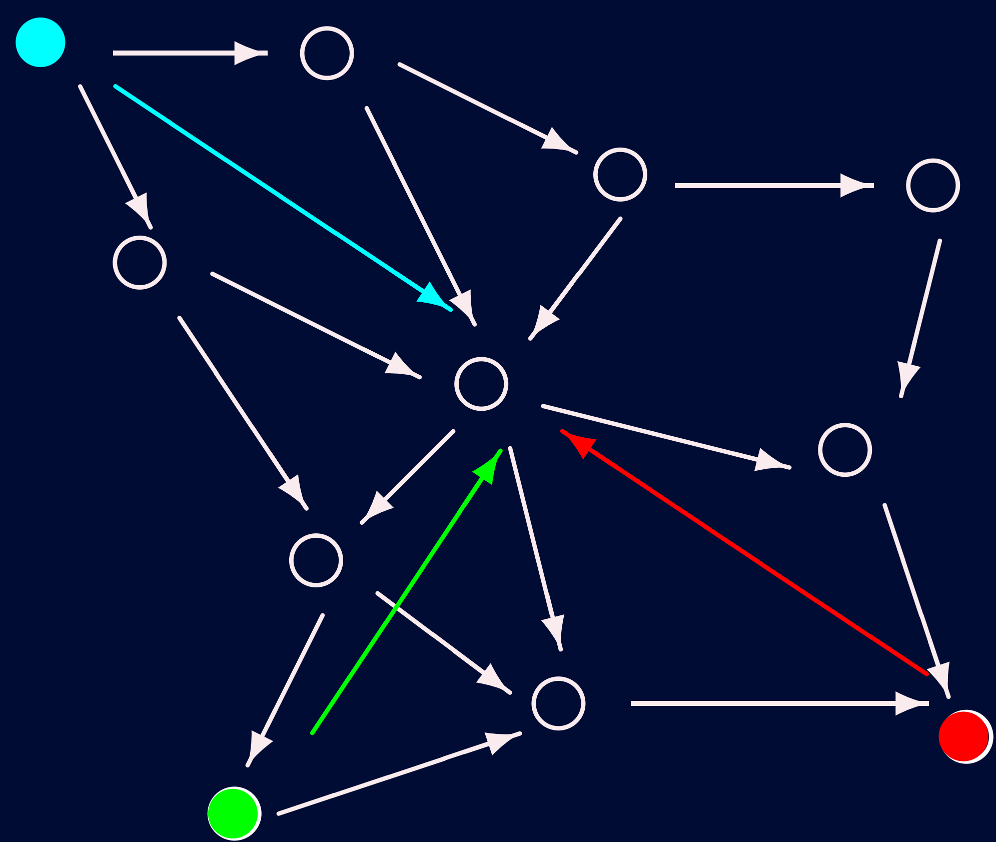

APS example

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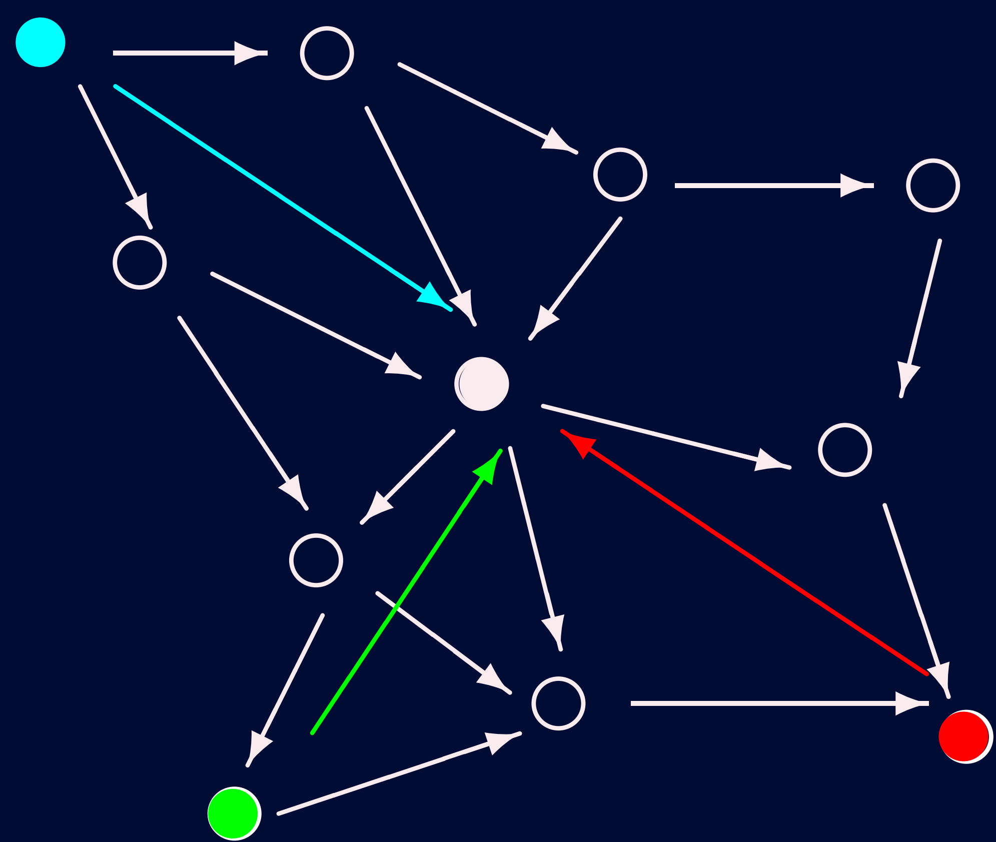

DV-hop propagation

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- standard DV propagation
- each <u>node</u> maintains a table $\{X_i, Y_i, h_i\}$
- each <u>landmark</u> $\{X_i, Y_i\}$
  - computes a correction $c_i = \frac{\sum \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}}{\sum h_i}$
  - ...and floods it into the network
- each node
  - uses the correction from the closest landmark
  - multiply its hop distances by the correction

DV-hop error analysis

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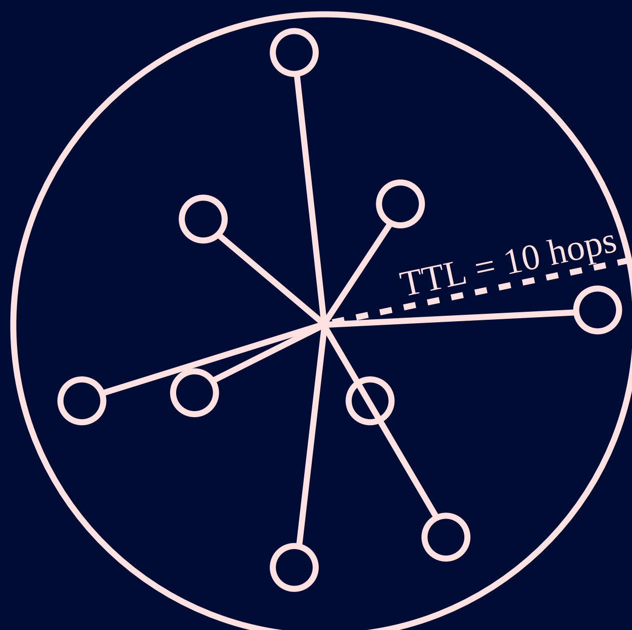
Q: how good are the obtained positions?

A: it depends on: density, landmark ratio, TTL.

Error CRLB derivation:

1. **range error → trilateration error lower bound**
2. **approximate range error for *DV-hop***
3. **integrate over all landmarks**

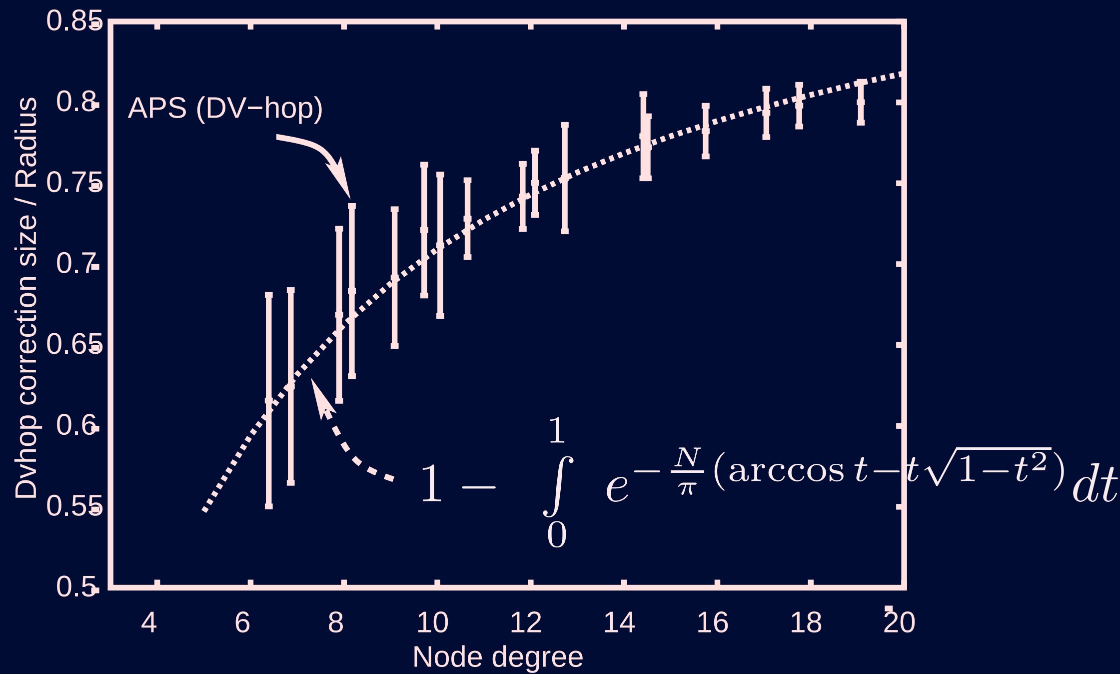


trilateration positioning error ☞

- x_i = coordinates of landmark i
- x = true position
- ρ_i = distance to landmark i
- assumptions - ρ_i are
  - normal $\rightarrow$ covariance W
  - independent
- position is a parameter
- Jacobian $J_0 = \frac{\mathrm{x}_i - \mathrm{x}}{\rho_i}$
- **CRLB(Cov[position]) =** $\left(J_0^\top W J_0\right)^{-1}$

DV-hop range error

- shortest path from node to landmark
- how many hops?
- estimate progress in one hop z

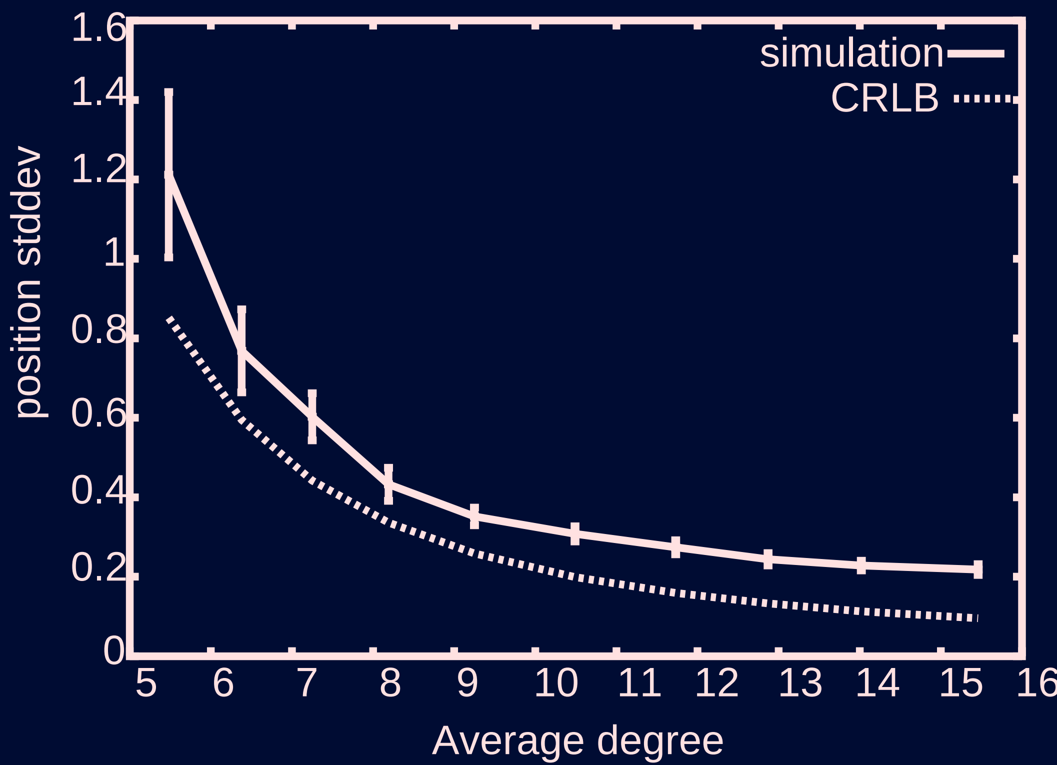


- **range error variance $\sim$ number of hops**

DV-hop position error

$$CRLB(cov[x\ y]) = \frac{1}{f\pi h}\frac{V[z]}{\lambda E^2[z]}I_2$$

- f = **landmark ratio**
- h = **TTL in hops**
- $\lambda = \frac{degree}{\pi}$ = **density**
- z = **progress per hop**
  - **depends on** λ
  - **no closed form**



DV-position

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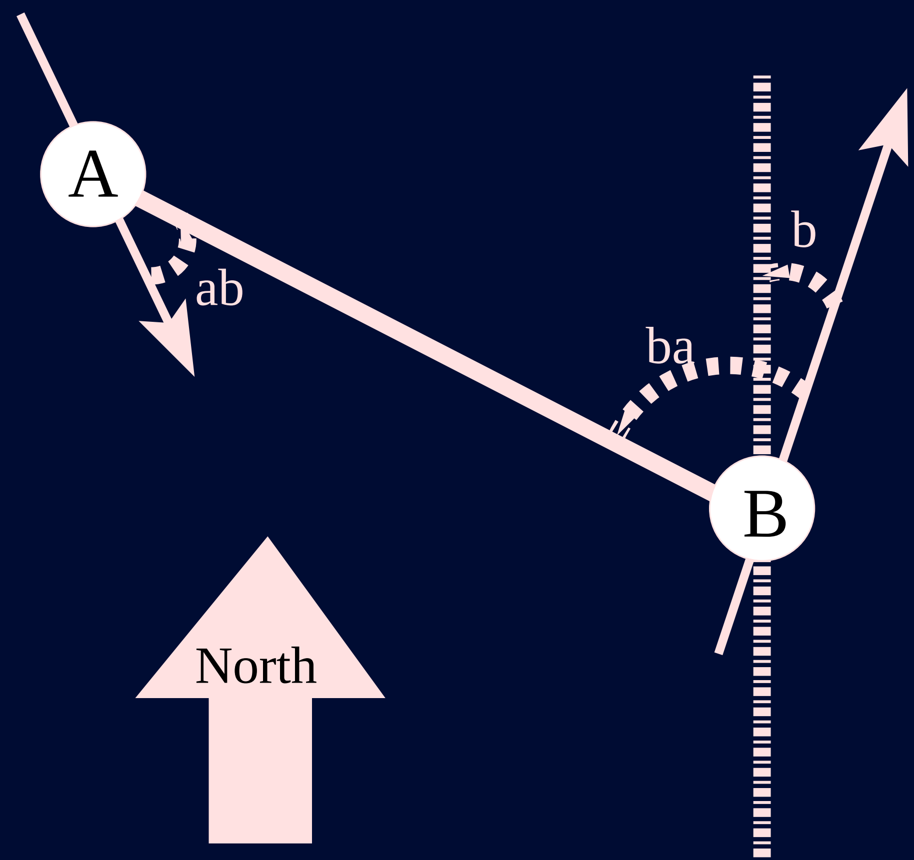
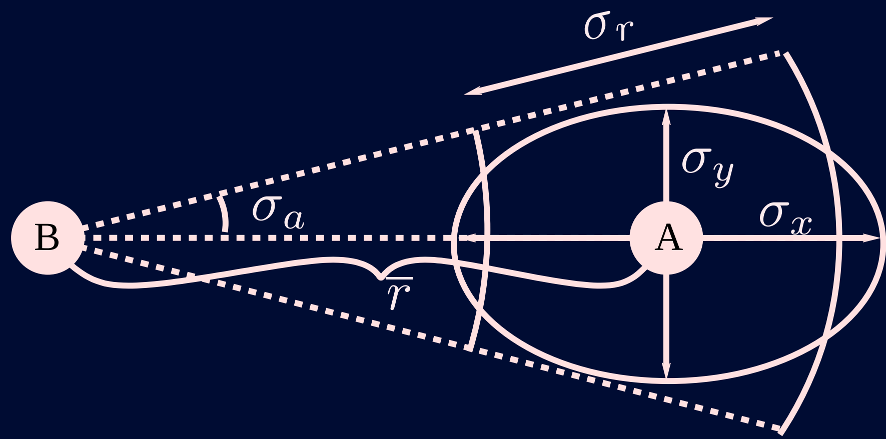


- needs **both angle and range** measurements
- need compasses at least at landmarks
- one hop positioning

DV-position error



3. **combine landmarks with Kalman filter**
4. **when** $\sigma_a = \sigma_r = \sigma$

$$Cov = \frac{1}{2f\pi h}\sigma^2 \frac{E[r^2]}{\lambda E^2[z]} I_2$$

1. **approximate one hop error**
2. **cumulate error along path to landmark**

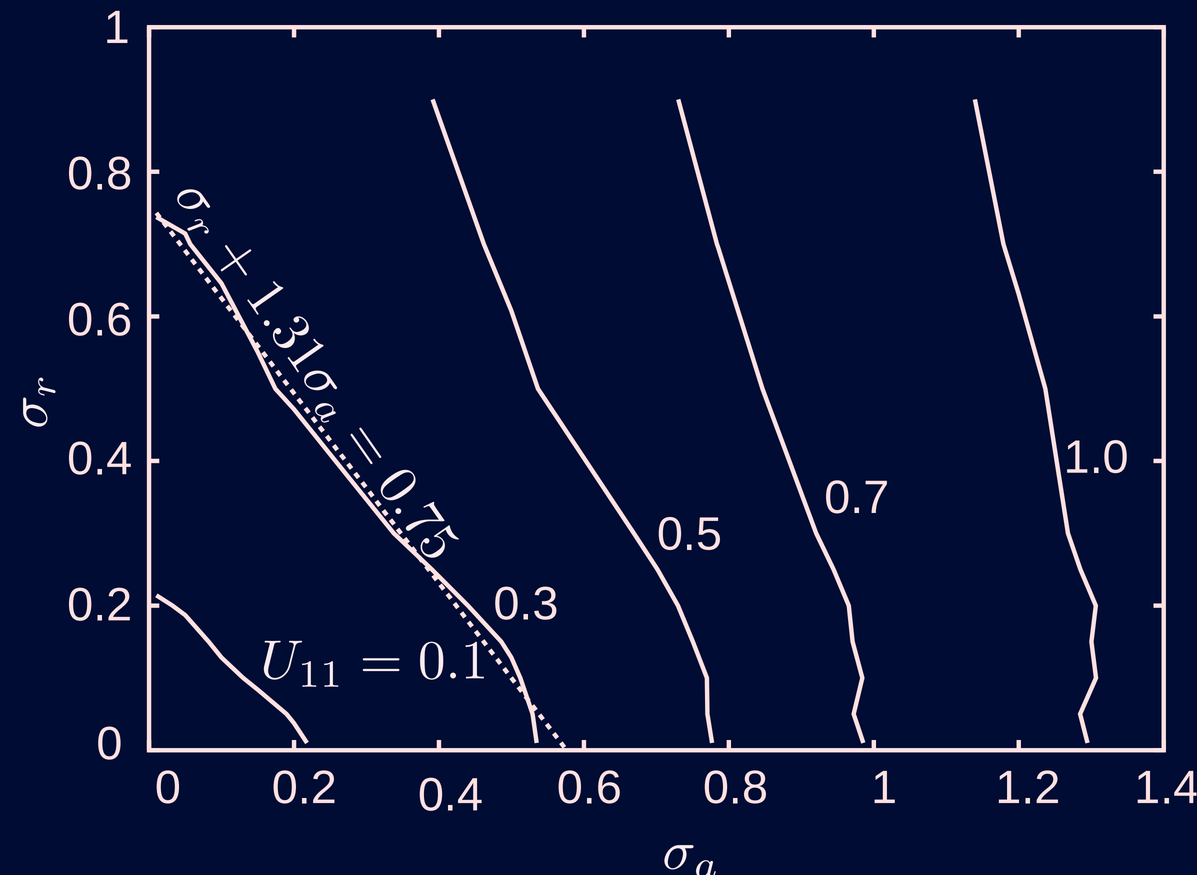
$$U_A = U_B + R^\top \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} R$$

DV-position

Simulation: 10000 nodes, $f = 1\%,\ \lambda = \frac{10}{\pi},\ TTL = 15$

σ_a **= standard deviation in angle error** $(0.1 \simeq 5.7^o)$

σ_r **= standard deviation in range error**



range only / angle only

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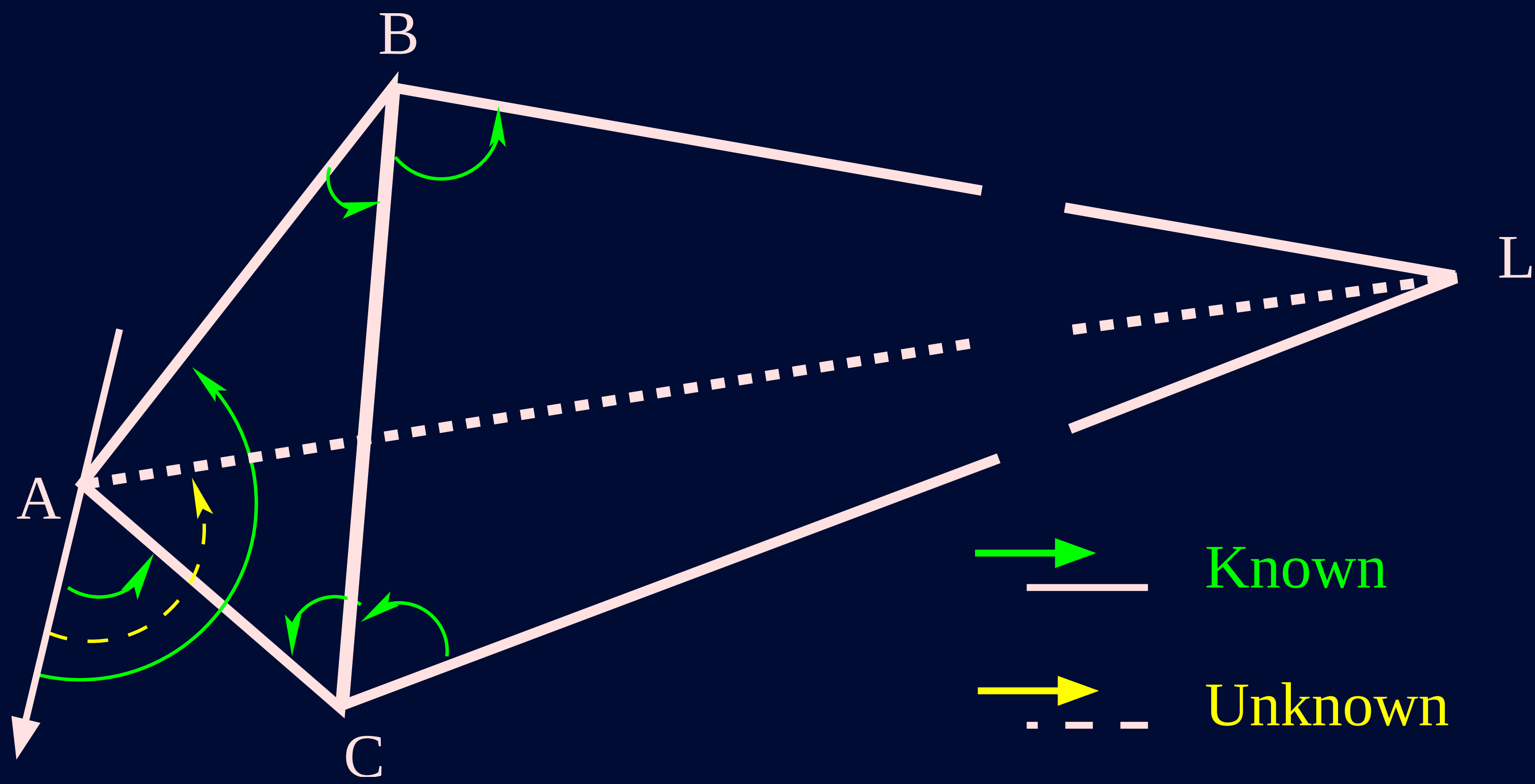

- *DV-radial* → find bearing to L (yellow angle)
- *Euclidean* → find range to L (distance AL)

parameter space

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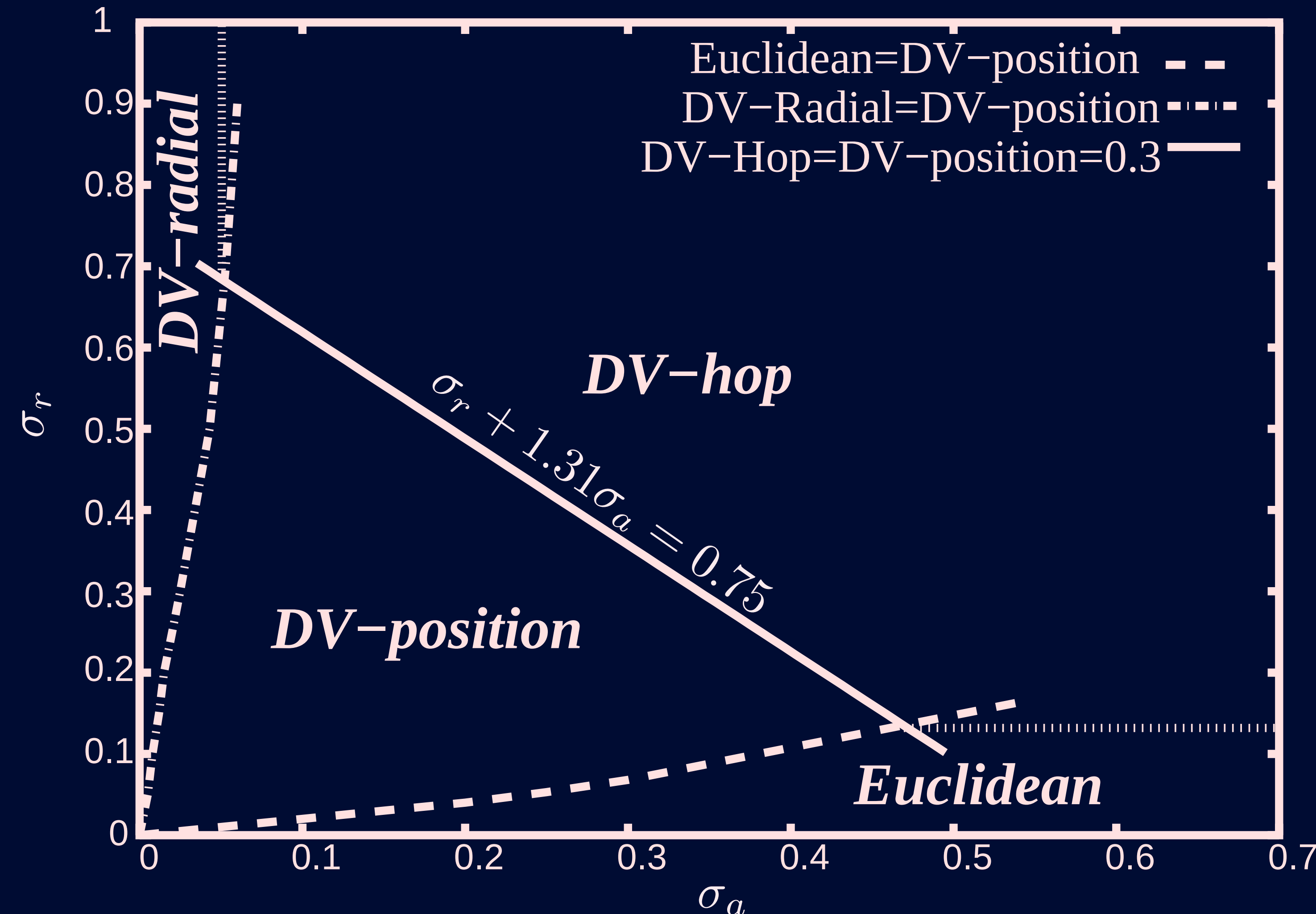

future work

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- status
  - *DV-hop* implemented on motes
- node mobility
  - a moving landmark
    - is a new landmark
  - mobile nodes are supported by static nodes
  - on demand positioning?
- error analysis
  - *DV-bearing* (angle based)
  - *Euclidean* (range based)
- comparison to centralized schemes

APS summary

- Ad Hoc Positioning System (APS)
  - DV based positioning
  - infrastructure free
  - distributed, localized, multihop
  - wide range of capabilities
- APS accuracy complex tradeoff
  - hardware capabilities
  - network density, landmark ratio, TTL
  - quality of positions
- only good hardware is better than “no hardware”

extra

- simulation
  - *Euclidean, DV-hop*
  - DV-radial
  - tracking
- DV-hop error
- AoA nodes
- error control
- trilateration
- triangulation
- V.O.R.

simulation

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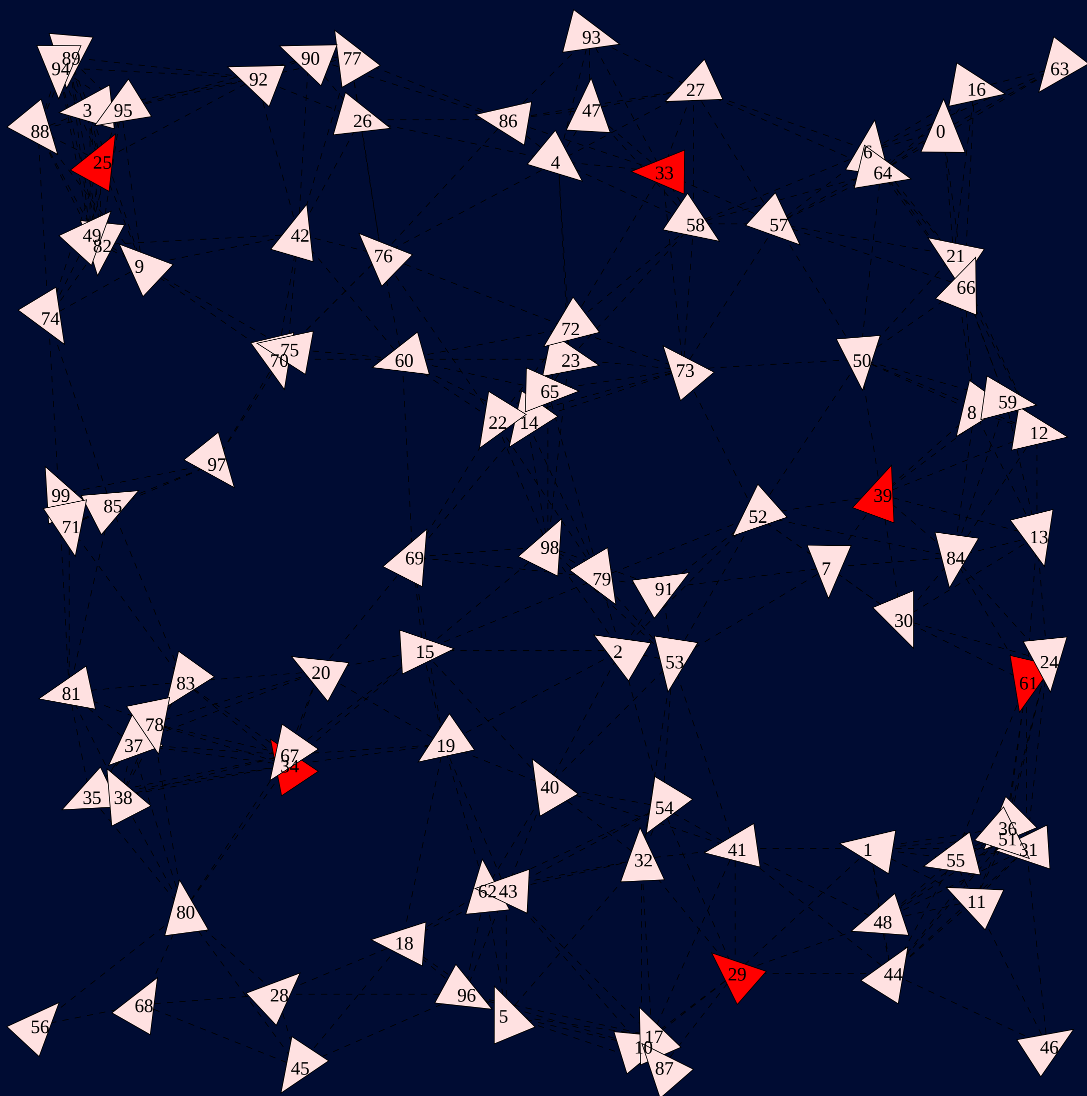

1000 nodes, degree≃10.5, random unknown heading, white noise AoA measurements

position error
range based, range free

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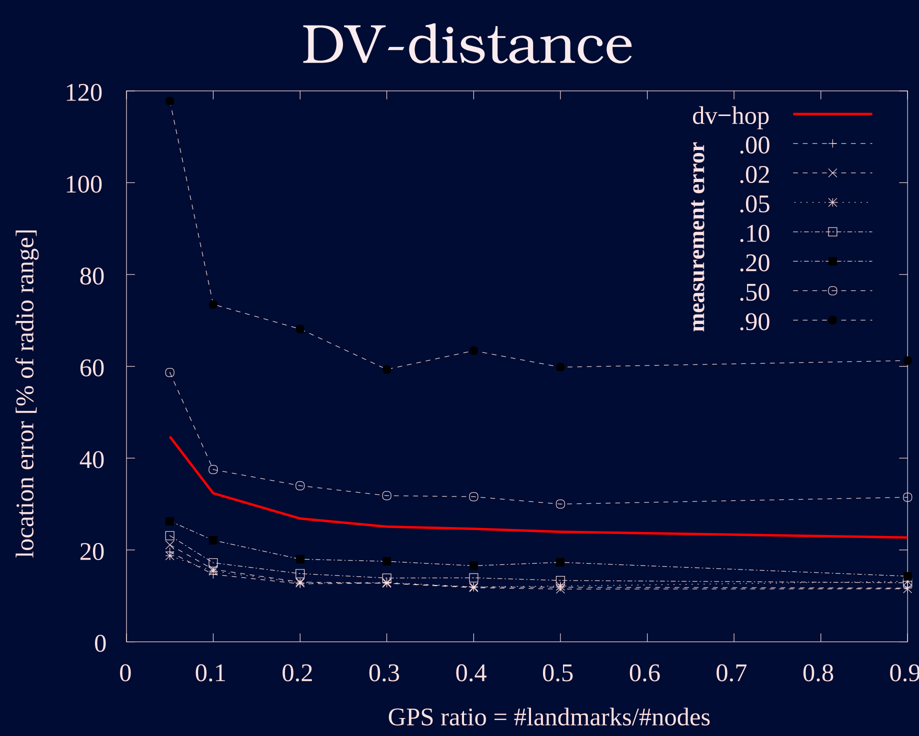
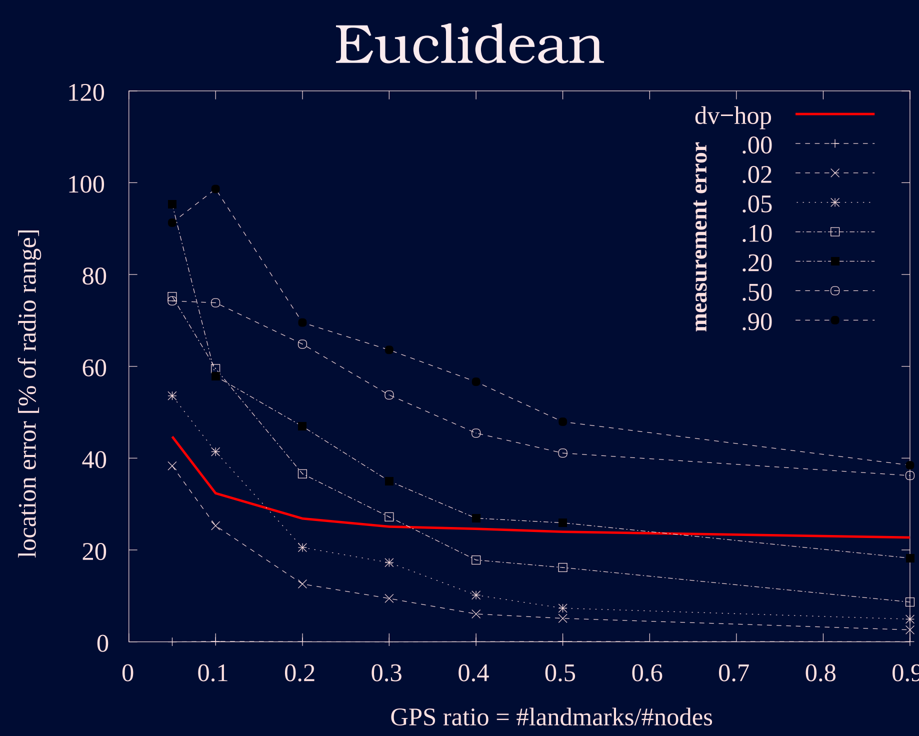
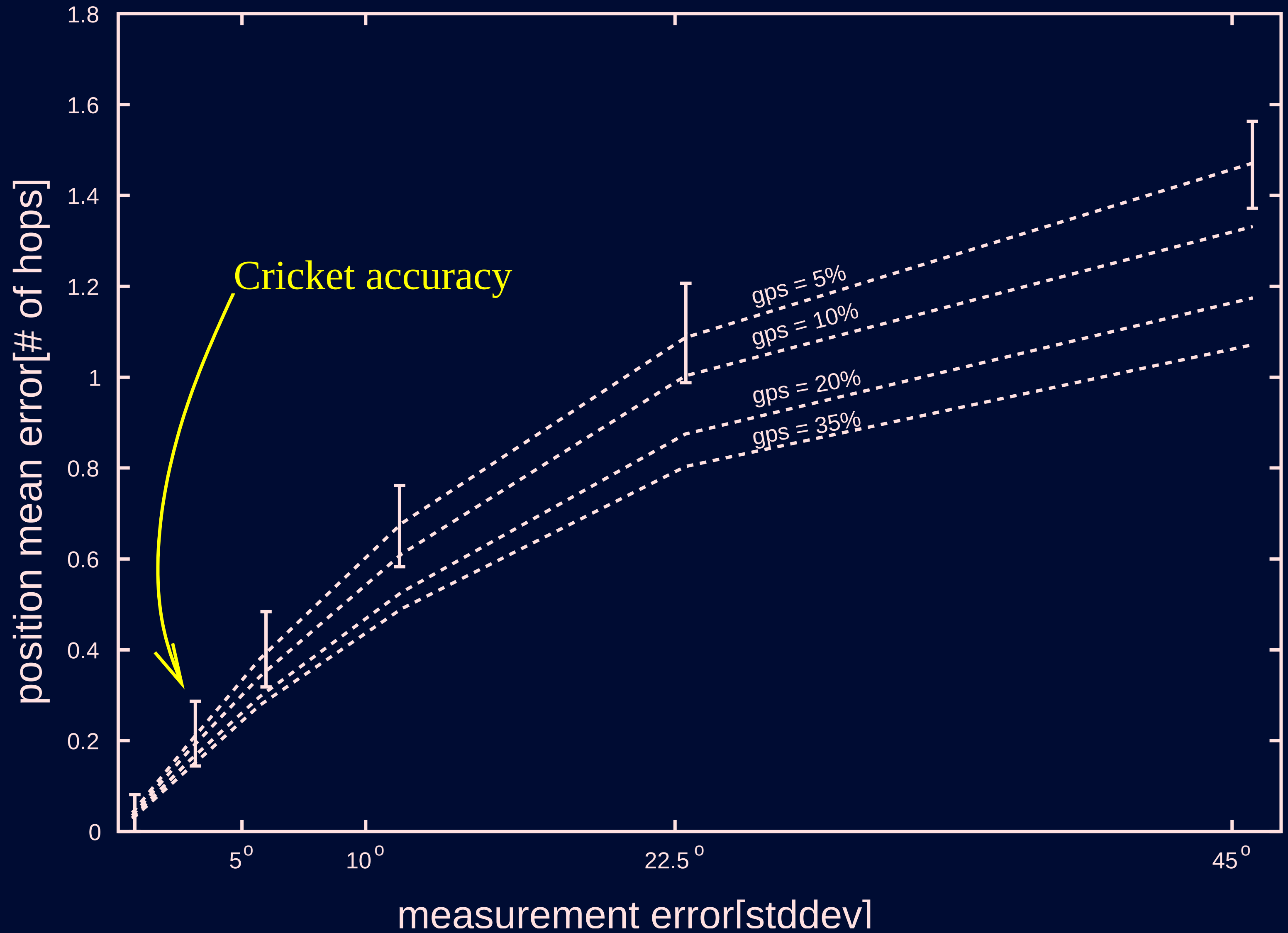
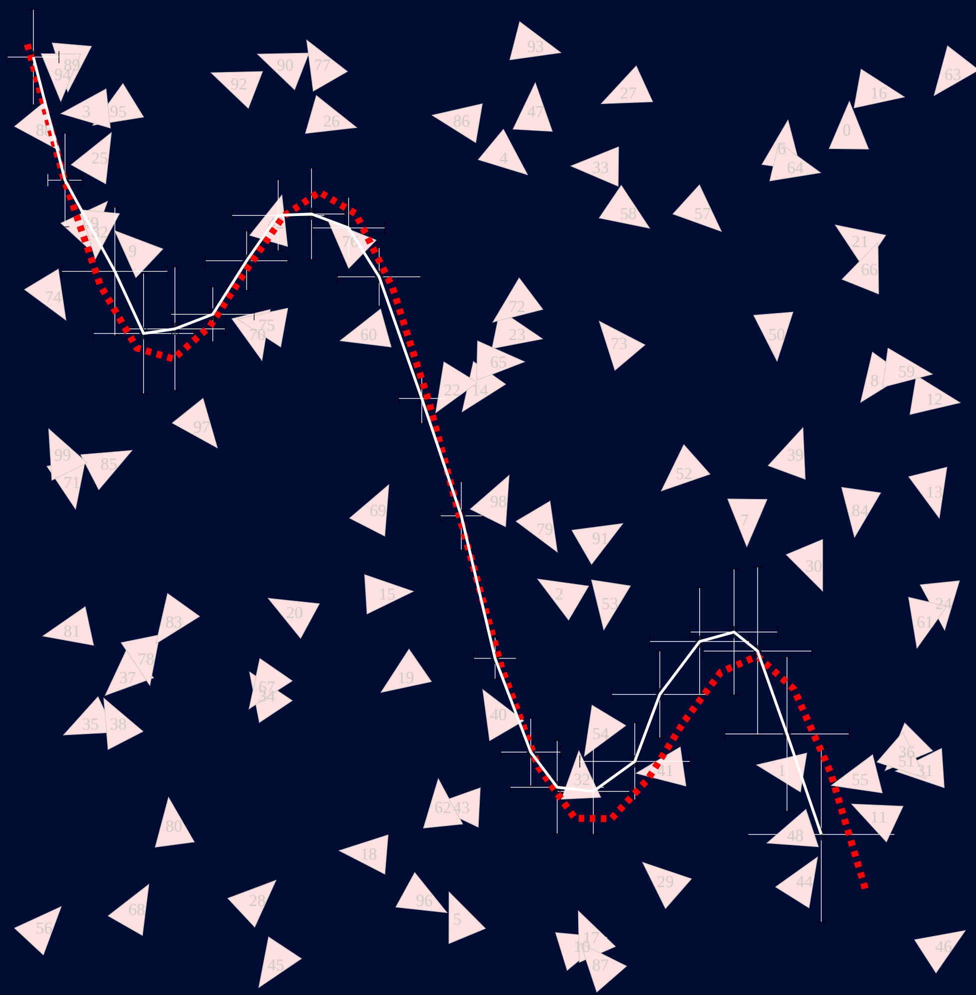




position error - AoA based ☞



tracking example (AoA)



DV-hop error analysis

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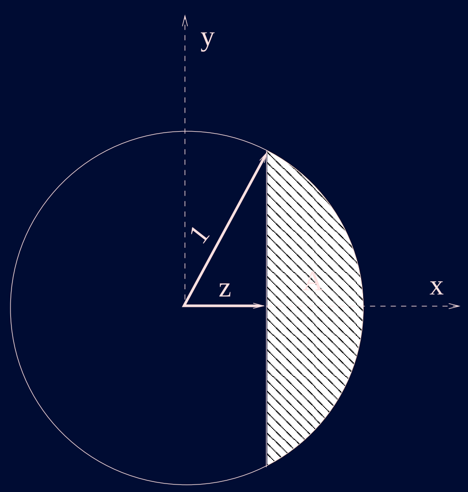
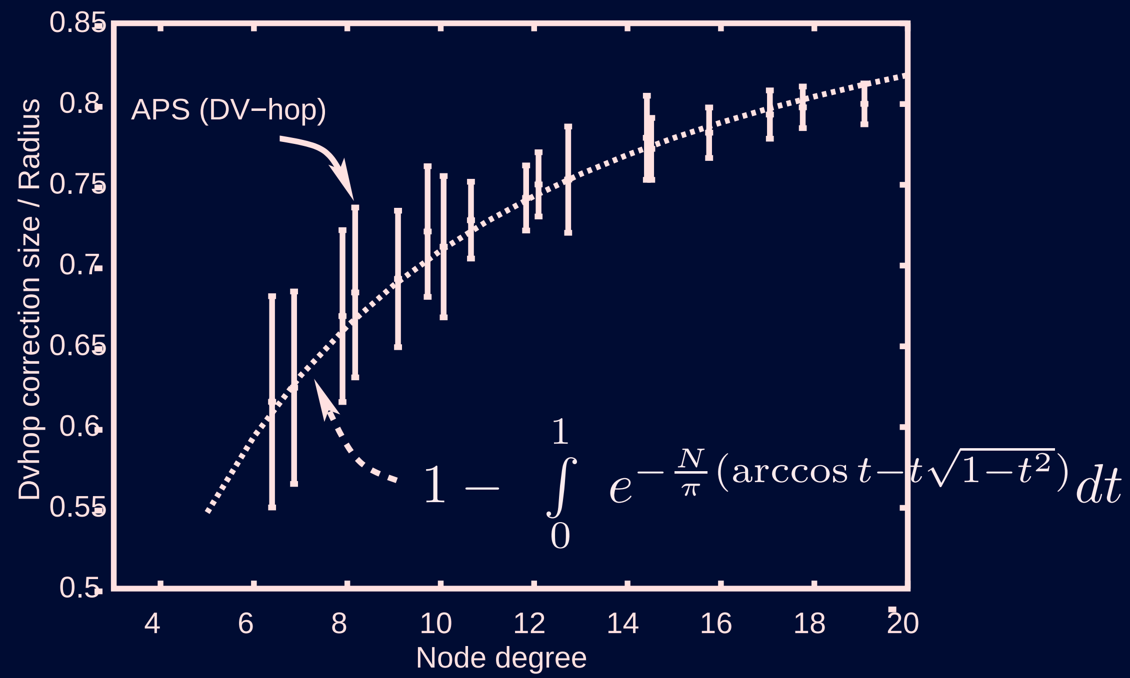
- MFR → shortest path
- $\bar{z}, \sigma_z$ expectation, deviation of MFR advancement

$$\bar{z} = 1 - \int_0^1 e^{-\frac{N}{\pi}(\arccos t - t\sqrt{1-t^2})} dt$$

$$\sigma_z = \ldots$$

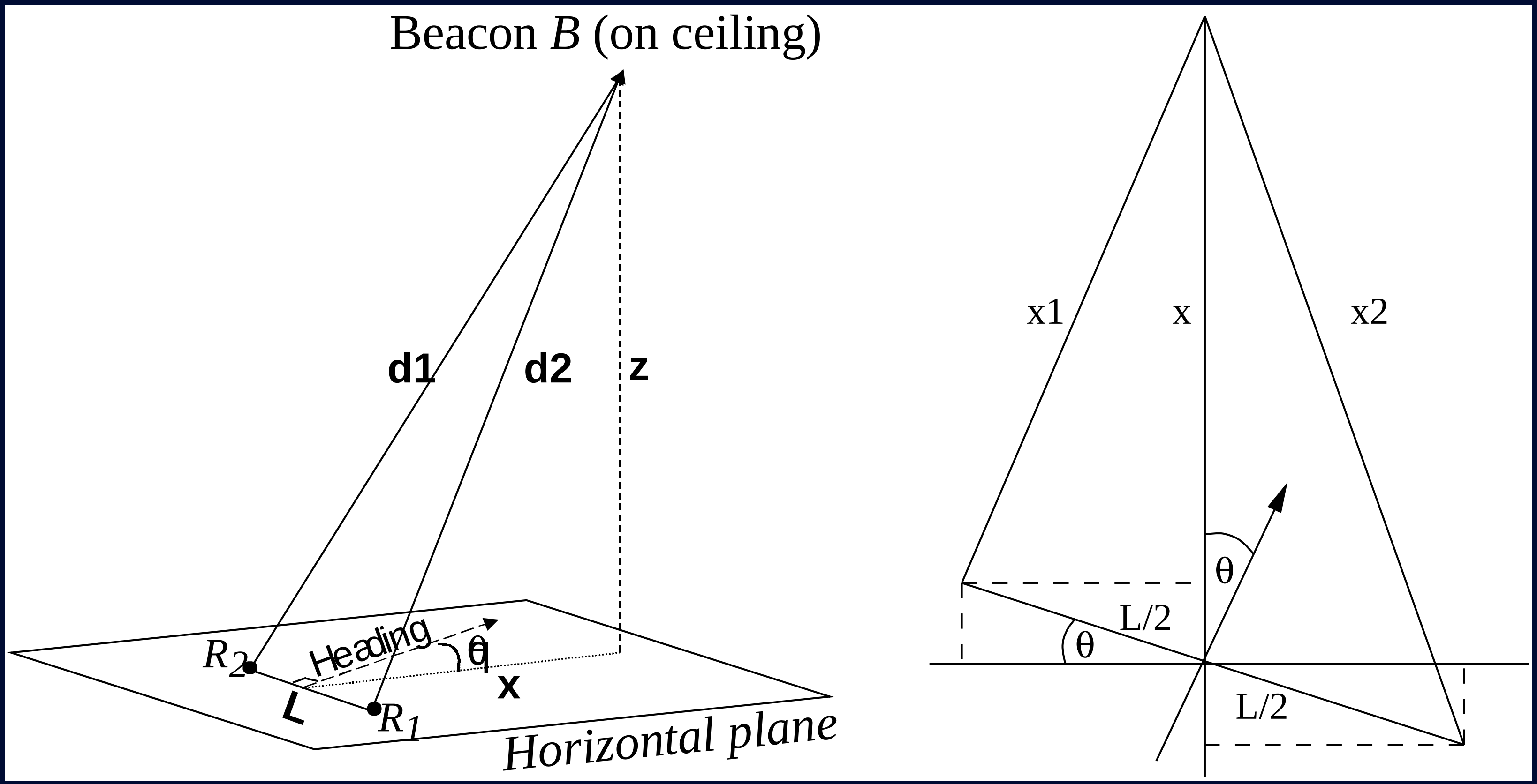




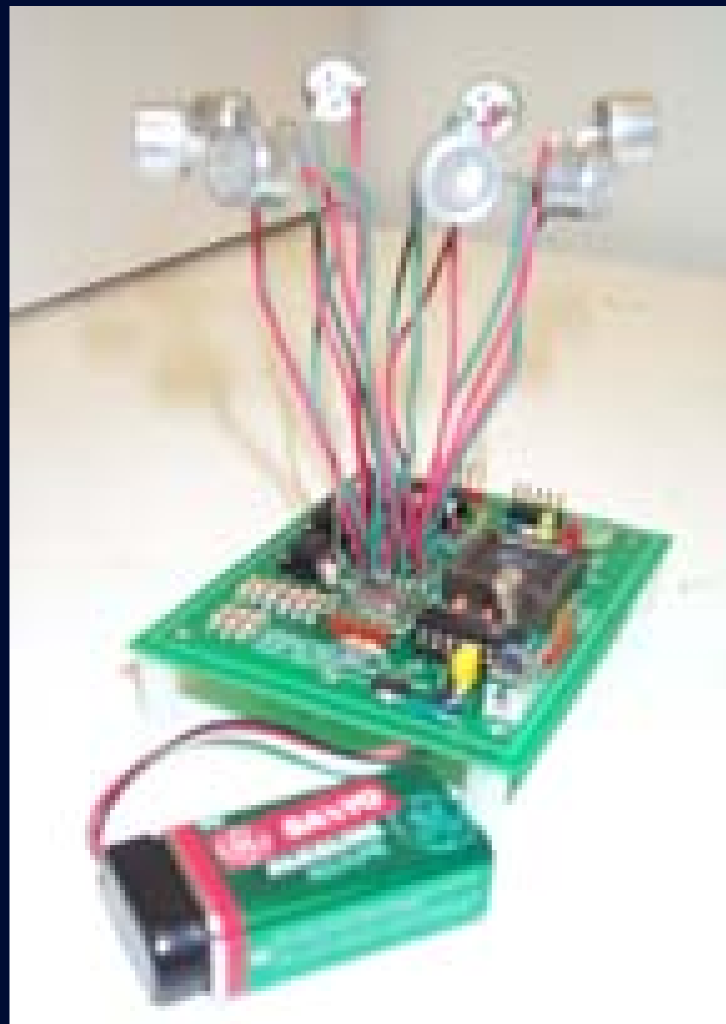
angle of arrival capable nodes ☞

- **Cricket** compass - [Pryiantha01]
  - uses 5 ultrasound receivers
    - 0.8cm each
    - a few centimeters apart
  - TDoA
  - $\pm 10\%$ accuracy for angles < 40 degrees
- **Medusa** node - [Savvides2001]

Cricket - basic principle



Medusa node



trilateration

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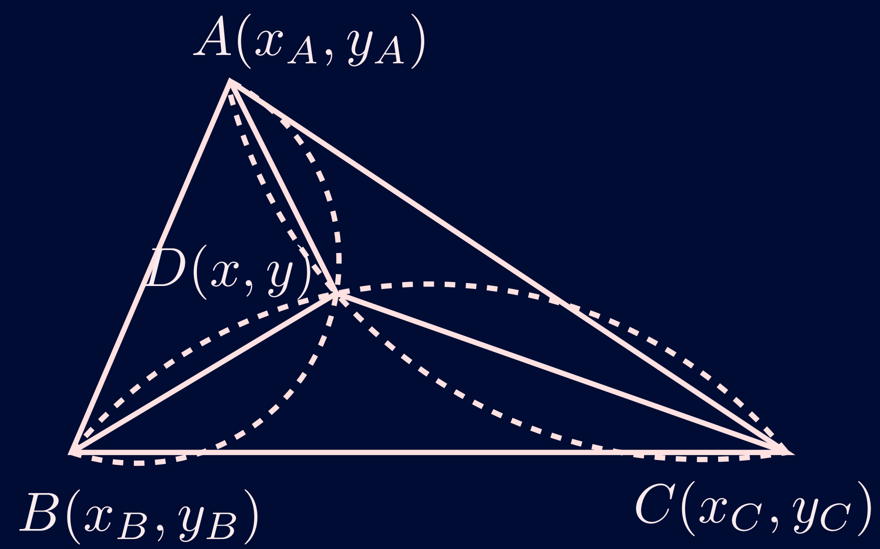
$$
\begin{aligned}
(x - x_i)^2 + (y - y_i)^2 &= (d_i + \epsilon_i)^2 \\
x_i,\ y_i &\rightarrow landmark\ coordinates \\
d_i &\rightarrow measured\ ranges
\end{aligned}
$$

- used in GPS
- can be linearized
- least squares not good for large outliers
- use weights → variance of distances

triangulation

- Given
  - positions of the landmarks (x_i, y_i), $i = A, B, C$
  - bearings to landmarks $\longrightarrow$ $A\widehat{D}B$, $B\widehat{D}C$, $C\widehat{D}A$
- a node may infer its:
  - own position (x, y)
  - bearing to North
- intersection of circles



triangulation

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- each node obtains a table $\{X_i, Y_i, dir_i\}$ with coordinates and bearings to landmarks
- how to find position?
  - $\binom{n}{2}$ pairs of landmarks
    - intersect circles → nonlinear system
    - distances to centers → GPS problem
  - $\binom{n}{3}$ triplets of landmarks → $\binom{n}{3}$ estimates → centroid
  - $O(n)$ algorithm [Betke94] → same accuracy
- how to find absolute orientation?
  - position + bearing to known point

V.O.R.

$$\begin{aligned} a_i x + b_i y &= c_i \\ if\ cos(\alpha_i) &= 0 \\ & a_i = 1 \\ & b_i = 0 \\ & c_i = x_i \\ else & \\ & a = \tan(\alpha_i) \\ & b = -1 \\ & c_i = -y_i + x_i \tan(\alpha_i) \end{aligned}$$